NONHYDROSTATIC ADIABATIC KERNEL FOR HIRLAM
Part I
Fundamentals of nonhydrostatic dynamics in pressure–related coordinates

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1 Introduction

We start with the description of a numerical model of atmospheric dynamics, which is designed as the nonhydrostatic extension to the adiabatic kernel of the present hydrostatic HIRLAM. For representation of nonhydrostatic dynamics, the model makes use of a pressure coordinate framework, and numerical codes are formulated, like in the hydrostatic parent-HIRLAM, in pressure-based hybrid coordinates of ECMWF origin.

The pressure-coordinate approach to nonhydrostatic modeling has many advantages in comparison with other methods. Besides generality – pressure coordinates support presentation of most general, non-simplified dynamics – this approach does not require prior separation of the background state, inherent in many nonhydrostatic models. Another advantage is simplicity – in pressure coordinates any optional atmospheric motion with sophisticated mass-distribution becomes an incompressible flow with constant density. A practical advantage of pressure-coordinate approach in nonhydrostatic modeling is that it supports immediate generalization of the existing hydrostatic HIRLAM into the domain of nonhydrostatic spatial scales. Numerical packages, such as initialization tools, sub-grid physical parametrization libraries, and postprocessing facilities, can be maintained. Due to the enhanced resolution of nonhydrostatic dynamics, initialization and physical parametrization must be revised, but there is no need to start from the beginning. Thus, the overall development cost may be significantly reduced.

The first NH model in pressure coordinates was developed by Miller (1974), and Miller and Pearce (1974). That was a pure pressure-coordinate model with the background separation and accompanying linearization of energy conversion term. The Miller-Pearce model is pseudo-anelastic ("partially anelastic") – internal acoustic waves are filtered due to conservation of pressure-coordinate volume of air particles, yet the external acoustic mode (Lamb wave) is maintained. A modification of the Miller-Pearce model to a sigma–coordinate framework was given by Miller and White (1984). White (1989) proved the existence of a more general model, which does not require background separation and linearization, and provided a sigma-coordinate representation of that model. A variational formulation of the White model was given by Salmon and Smith (1994). Rõõm (1997) developed a completely anelastic sigma–coordinate model, in which surface pressure is adjusted and the Lamb mode filtered. Numerical formulations of the pseudo-anelastic model have been developed by Xue and Thorpe (1991), Miranda and James (1992), and for the anelastic model by Rõõm (1997).

A different (parallel to the above–cited development) pressure-coordinate representation theory of atmospheric dynamics was given by Rõõm (1989, 1990, 1998), who showed that exact (acoustically non-filtered) dynamics can be presented in pressure-related coordinates, if vertical accelerations do not exceed gravitational acceleration. In that approach the main metrical relations and continuity of the pressure-coordinate space are first explicitly defined and the continuity equation for pressure-space density is introduced, which closes the system of dynamic equations. The variational formulation of general equations is given in (Rõõm 1999). An important quality of the general approach is that it includes all simplified pressure-coordinate approaches, like hydrostatic, pseudo-anelastic, and anelastic models, as special cases, which allows for a unified treatment of all pressure-coordinate dynamics. This generality was employed by Rõõm and Männik (1999) at unified testing of accuracy of different pressure-coordinate models.

The paper is designed and realized in two parts, each representing a separate chapter:

- Part I. Fundamentals of nonhydrostatic atmospheric dynamics in pressure–related coordinates.
- Part II. Anelastic, hybrid-coordinate, Explicit-Eulerian model.

Part I presents the fundamentals of general atmospheric dynamics in pressure coordinates. The problem is, that in spite of simplicity of pressure-coordinate dynamics, its interpretation is not just as trivial as in the hydrostatic case and can cause ambiguities, if not preliminarily separated into elementary pieces. Though the numerous papers, cited above, deal with the main aspects of the pressure-coordinate models in many details, we still lack an elementary concise presentation of the basic postulates of the theory. It appears that a paper like this, designed for wide use as an introductory documentation into the numerical nonhydrostatic model based on a pressure coordinate presentation, is just the right place for a careful description of the basics of the approach. The discussion of the main facts is illuminating, helps to avoid ambiguities, and makes further representation more simple, transparent and concise. In this chapter, initial definitions are presented in the framework of general pressure-coordinate dynamics, followed by tracing the changes, which can appear when simplifications like anelasticity are introduced. For instance, the pressure coordinate itself serves as an example: is that the actual pressure, or just its hydrostatic component, and what happens to the pressure when the anelastic approximation is applied? Those familiar with nonhydrostatic dynamics in pressure-coordinates can easily omit this chapter, and start from the second part, which introduces the final anelastic hybrid-coordinate model.

2 Exact equations of adiabatic atmospheric dynamics in pressure coordinates

2.1 Pressure vertical coordinate

In pressure coordinates the pressure p of an air particle is treated as the vertical coordinate of this particle and it determines along with horizontal coordinates $x,\ y$ the location (or $p-$ *coordinates*) $\{x, y, p\}$ of the particle in the *pressure coordinate system*. Consequently, the components of the velocity of the material particle in p-coordinates are $\{u,\ v,\ \omega\}$

$$u = \frac{\mathrm{d}x}{\mathrm{d}t}\ , \quad v = \frac{\mathrm{d}y}{\mathrm{d}t}\ , \quad \omega = \frac{\mathrm{d}p}{\mathrm{d}t}\ , \tag{2.1.1}$$

and the material derivative of function A is

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + \omega\frac{\partial A}{\partial p}\ . \tag{2.1.2}$$

Ordinary height becomes in the pressure coordinate system a function of p-coordinates and time, $z = z(x, y, p, t)$, and it is often called the *isobaric height*.

In the moving atmosphere, the pressure consists of the hydrostatic component p^s and the nonhydrostatic contribution p^n

$$p = p^s + p^n\ , \tag{2.1.3a}$$

where p^s is determined via the barometric relation for any given height z

$$p^s = g\int_z^\infty \rho dz'\ , \tag{2.1.3b}$$

ρ being the air density, and the nonhydrostatic part is defined as the difference between pressure and hydrostatic pressure

$$p^n = p - p^s\ . \tag{2.1.3c}$$

When the atmosphere is in hydrostatic equilibrium, then $p^n = 0$, and the pressure coordinate coincides with the hydrostatic pressure of the particle, but this is not the general case, and in the moving atmosphere p and p^s must be strictly distinguished. In the present study, the vertical coordinate is always identified with the actual pressure p.

2.2 Continuity

The continuous quality of the medium is specified via the density n of the matter in pressure coordinates and in the corresponding continuity equation. We define nondimensional density via the mass dm of an elementary air volume:

$$dm = \rho|dxdydz| = n|dxdydp|/g \ ,$$

where g is the gravitational acceleration. Keeping in mind, that a negative dp corresponds to the positive dz, and using the general gas law, we get

$$ndp = -g\rho dz = -(p/H)dz \ , \tag{2.2.1}$$

where

$$H = p/(g\rho) = RT/g \tag{2.2.2}$$

is the local *scale height*, T is the temperature and R is the gas constant of the air. The differential equality (2.2.1) yields a *metric equation*

$$\frac{p}{H}\frac{\partial z}{\partial p} = -n \ . \tag{2.2.1$'$}$$

For hydrostatic equilibrium conditions, $p \to p^s$, the density n becomes equal to 1 and the metric equation simplifies to the ordinary hydrostatic equation. The density n satisfies the *continuity equation* (representing the mass conservation law in pressure coordinate system)

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) + \frac{\partial n\omega}{\partial p} = 0 \ , \tag{2.2.3}$$

or in an equivalent form

$$\frac{\mathrm{d}n}{\mathrm{d}t} + n\left(\nabla \cdot \mathbf{v} + \frac{\partial \omega}{\partial p}\right) = 0 \ , \tag{2.2.3$'$}$$

where

$$\nabla = \mathbf{i^x}\frac{\partial}{\partial x} + \mathbf{i^y}\frac{\partial}{\partial y} \ , \qquad \mathbf{v} = \mathbf{i^x}u + \mathbf{i^y}u$$

are horizontal gradient in pressure coordinates and horizontal wind vector, whereas $\mathbf{i^x}$, $\mathbf{i^y}$, $\mathbf{i^z}$ are unit vectors along x, y, and vertical axes. The continuity equation (2.2.3) along with the metric equation (2.2.1') determine the quality of the curvilinear pressure-coordinate space.

The density n can be used for hydrostatic pressure diagnostics. From (2.1.3b) and (2.2.1) a relationship follows:

$$p^s = \int_o^p n dp \ , \tag{2.2.4}$$

or, in the differential form

$$\frac{\partial p^s}{\partial p} = n \ . \tag{2.2.4'}$$

2.3 Equations of motion and thermodynamics

The equations of motion and thermodynamics in pressure coordinates, in adiabatic and friction-free case, are as follows (Rõõm 1989, 1998):
Isobaric height equation

$$\frac{\mathrm{d}z}{\mathrm{d}t} \ = \ w \ ; \tag{2.3.1}$$

Vertical momentum equation

$$n \, \frac{\mathrm{d}w}{\mathrm{d}t} \ = \ g(1 \ - \ n) \ ; \tag{2.3.2}$$

Horizontal momentum equations

$$n \, \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \ = \ - \ g \nabla z \ - \ n \mathbf{f} \times \mathbf{v} \ ; \tag{2.3.3}$$

Temperature equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} \ = \ \frac{\varkappa T \omega}{p} \ , \tag{2.3.4}$$

where w is vertical velocity, $\mathbf{f} = f \mathbf{i^z}$, and f is the Coriolis parameter, $\varkappa \ = \ R/c_p$ and c_p is isobaric specific heat. It is assumed that the atmosphere follows the equation of the state of an ideal gas (this assumption was already used at the definition of height scale H (2.2.2)).

Seven equations (2.2.1'), (2.2.3), (2.3.1) - (2.3.4) present a closed system for determination of seven fields n, z, u, v, w, T and ω.

2.4 Domain and boundary conditions

The conditions at lateral boundaries are the same as in Cartesian coordinate models (and are not treated here, though we will discuss them later while specifying the elliptic equation's boundary conditions). Differences occur in the "horizontal" conditions at the top and at the bottom. The domain occupied by the atmosphere is

$$0 \ < \ p \ < p_0(x, y, t) \ , \tag{2.4.1}$$

where the lower boundary p_0 evolves in accordance with the equation

$$\frac{\mathrm{d}p_0}{\mathrm{d}t} \ = \ \omega|_{p_0} \ , \tag{2.4.2}$$

expressing the condition that the lower boundary consists of the same air particles all the time. Thus, the domain is varying in time and (2.4.2) presents an additional prognostic equation which must be integrated along with the remaining system.

The boundary condition at $p \ = 0$ is

$$\lim_{p \ \to \ 0} \omega \ = \ 0 \ . \tag{2.4.3}$$

and it forbids the outflow of mass into outer space.

The primary boundary condition at the underlying surface is

$$z|_{p_0} \ = \ h(x, y), \tag{2.4.4}$$

where $h(x, y)$ represents the surface elevation above sea-level.

Like p is not hydrostatic in the general model, the surface pressure p_0 is different from the hydrostatic surface pressure component p_0^s. A diagnostic equation for p_0^s follows from (2.2.4):

$$p_0^s(x, y, t) \equiv p^s[x, y, p_0(x, y, t), t] = \int_0^{p_0(x,y,t)} n(x, y, p, t) dp \ . \tag{2.4.5}$$

Action on this relationship with $\partial/\partial t$ yields (with the help of (2.2.3), (2.4.2) and (2.4.3))

$$\frac{\partial p_0^s}{\partial t} + \nabla \cdot \int_0^{p_0} n\mathbf{v} dp \ = \ 0 \ . \tag{2.4.6}$$

This is the tendency equation for hydrostatic component of the surface pressure in nonhydrostatic pressure-coordinate dynamics. It expresses the vertically integrated mass conservation law.

2.5 Isobaric height components

From (2.2.1') and (2.4.4), an integral representation follows for isobaric height

$$z = h + \int_p^{p_0} nH \frac{dp}{p} \ .$$

For applications it is practical to split z between a hydrostatic part z^* and another contribution $\tilde{z}$

$$z = z^* + \tilde{z} \ ,$$

where z^* is a solution of the hydrostatic equation

$$\frac{p}{H}\frac{\partial z^*}{\partial p} = -1 \ . \tag{2.5.1}$$

The general solution of this equation is

$$z^* = h(x,y) + \int_p^{p^*} \frac{H}{p'} dp' \ , \tag{2.5.2}$$

where p^* stands for the constant of integration. Different choices of p^* yield different z^* and $\tilde{z}$. The most important choices of p^* are the hydrostatic component of surface pressure p_0^s, actual surface pressure p_0, and mean surface pressure $\overline{p}_0$.

In the case $p^* = p_0^s$ we will have splitting of the isobaric height into the hydrostatic and nonhydrostatic components

$$z = z^s + z^n \ , \tag{2.5.2a}$$

where

$$z^s = h(x,y) + \int_p^{p_0^s} \frac{H}{p'} dp' \ , \tag{2.5.2b}$$

and

$$z^n = z - z^s \ . \tag{2.5.2c}$$

Diagnostics of the hydrostatic component z^s from (2.5.2b) is a relatively sophisticated task, as it requires prior determination of p_0^s from (2.4.5).

More suitable for practical applications is the choice $p^* = p_0$, which leads to a splitting of z into *thermic* and *baric* components

$$z = z^t + z^b \ , \tag{2.5.3a}$$

$$z^t = h(x,y) + \int_p^{p_0} \frac{H}{p'} dp' \ , \tag{2.5.3b}$$

and

$$z^b = z - z^t \ . \tag{2.5.3c}$$

The most evident difference of two modes of splitting, (2.5.2) and (2.5.3), is that the thermic height equals the surface elevation height h at the lowest pressure level of the model, p_0, whereas the hydrostatic height becomes equal to the surface elevation at the hydrostatic surface pressure level p_0^s, which is different from the surface pressure. A relationship between the two representations is

$$z^t = z^s + \delta z \ , \qquad z^b = z^n - \delta z \ , \tag{2.5.4}$$

where

$$\delta z = \int_{p_0^s}^{p_0} \frac{H}{p'} dp' \approx \left(\frac{H}{p} \right)_{p_0} (p_0 - p_0^s) \tag{2.5.5}$$

is a barotropic (ie., height-independent) height shift.

The splitting is analogical when the mean surface pressure $\overline{p}_0$ is used instead of the actual pressure

$$z = z^t + z^b \ . \tag{2.5.6a}$$

Thermic and baric geopotential (we will use the same names as in previous case) are in this case

$$z^t = h(x,y) + \int_p^{\overline{p}_0} \frac{H}{p'} dp' \ , \tag{2.5.6b}$$

and

$$z^b = z - z^t \ . \tag{2.5.6c}$$

The choice of $\overline{p}_0$ is not unique. One possibility is the initial (analyzed) surface pressure distribution:

$$\overline{p}_0 = p_0(x,y,0) \ . \tag{2.5.7a}$$

Another possibility is presented, specifying $\overline{p}_0$ as a given (for instance, determined from a larger dynamical model), externally driven, time-dependent field:

$$\overline{p}_0 = \hat{p}_0(x,y,t) \ . \tag{2.5.7b}$$

Finally, $\overline{p}_0$ may be specified from the barometric formula

$$\overline{p}_0 = \tilde{p}_0(x,y) \equiv p_{sea} \exp\left[-\int_0^{h(x,y)} \frac{dz'}{H_0(z')}\right] \; , \qquad (2.5.7c)$$

where $H_0(z) = RT_0(z)/g$ is the horizontally averaged value of the scale height at altitude z, and p_{sea} is the mean sea-level pressure.

3 Anelastic pressure-coordinate model

The general pressure-coordinate dynamics presented in the previous chapter is too general and allows in the case of slow, large-scale motion several simplifications. The most straightforward simplified model – hydrostatic primitive-equation model – follows, if one puts $n = 1$ everywhere in the above-presented equations. However, pressure coordinate dynamics supports much more flexible approximations, which are almost as simple as the primitive equations, but maintain the nonhydrostatic nature of the model. In the following we will describe the *anelastic p-coordinate model*. This is a modification of the pseudo-anelastic model (White, 1989) with additional filtering of external mode sound waves (Lamb waves). The main attention will be paid to describing and commenting of simplifying assumptions.

3.1 Simplification of equations

i. The approximation $n = 1$ is applied everywhere, except for the metric equation (2.2.1') and the right hand term in the vertical momentum equation (2.3.2): n is equalized to the unit in the continuity equation (2.2.3) (or (2.2.3')), on the left hand of the equation (2.3.2) and in the equation (2.3.3) everywhere. In other words, approximation $n = 1$ is applied to the terms for which the density fluctuation $n' = n$ - 1 presents a second order small correction. As $n' << 1$ for slow dynamics at all spatial scales (typically $|n'| < 0.01$), this approximation is justified and maintains precision as long as quick processes (like explosions and shock wave propagation) are not involved.
ii. Equation (2.3.1) is approximated by the equation

$$w^s = -H\frac{\omega}{p} \qquad (3.1.1)$$

where w^s is a "hydrostatic" approximation of vertical velocity w; equation (3.1.1) is its definition. Consequent approximation of w by w^s is applied in the equation (2.3.2), which becomes (along with the approximation $n = 1$ on the left hand side)

$$\frac{\mathrm{d}w^s}{\mathrm{d}t} = g(1-n) \ + A_w. \tag{3.1.2}$$

Beginning with this equation and in what follows we have restored the diabatic forcing in prognostic equations. A_w represents here all terms omitted in simplified adiabatic treatment of vertical momentum equation: turbulent friction, spectral smoothing, etc.

An approximation for w similar to (3.1.1) was proposed already in the first formulation of nonhydrostatic p-space model by Miller and Pearce (1974), though they applied the linearized formulation with background scale height instead of the actual local scale H. Formula (3.1.1) was first introduced, and its harmony with energy conservation demonstrated, by White (1989). Approximation (3.1.1) can be justified by the following two-step simplification procedure. First, equation (2.3.1) is approximated as

$$w \approx \tilde{w} = \frac{\mathrm{d}z^s}{\mathrm{d}t} \ , \tag{3.1.3}$$

where z^s represents the hydrostatic isobaric height (2.5.2b). Secondly, on the right hand side of (3.1.3), which is written with the help of the material derivative definition (2.1.2) explicitly as

$$\frac{\mathrm{d}z^s}{\mathrm{d}t} \ = \frac{\mathrm{d}h}{\mathrm{d}t} + \left(\frac{H}{p}\right)_{p_0^s} \frac{\mathrm{d}p_0^s}{\mathrm{d}t} + \int_p^{p_0^s} \left(\frac{\partial H}{\partial t} + \mathbf{v}|_p \cdot \nabla H\right) \frac{dp'}{p'} - \frac{H}{p}\omega \ ,$$

the last term, as the dominant one at short spatial scales, is maintained only. At shorter spatial scales ($L_x <$ 10 -20 km), the first terms on the right hand side compensate each other, while the integral term is much smaller than the last one. These terms become comparable with the last term, $(-H\omega/p)$, in the hydrostatic region, yet there the pseudo-anelastic model behaves like an ordinary hydrostatic primitive equation model and does not depend on vertical velocity, and consequently, on its approximations. It should be noted that approximation (3.1.3) is also a valid one (see Rõõm 1999), yet it does not add much precision to the model performance in comparison with the (3.1.1), but makes computations more sophisticated (and, consequently, more noisy).

Another approximative definition for vertical velocity in the pseudo-anelastic model is proposed by Salmon and Smith (1994)

$$w \approx \hat{w} = -\frac{1}{g}\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t} \ , \tag{3.1.4}$$

where $\mathcal{H} = \mathcal{H}(p,s)$ represents the enthalpy, a characteristic thermodynamic function of pressure p and entropy density s . For a thermodynamically ideal gas, where

$$\frac{\partial \mathcal{H}}{\partial p} = \frac{1}{\rho} = \frac{RT}{p} \ , \frac{\partial \mathcal{H}}{\partial s} = T \ ,$$

definition (3.1.4) yields

$$\hat{w} = -\frac{H\omega}{p} - \frac{T}{g}\frac{\mathrm{d}s}{\mathrm{d}t} \ . \tag{3.1.4$'$}$$

For adiabatic processes $ds/dt = 0$, and (3.1.4') reduces to the original White definition (3.1.1). For practical applications, the same criterion holds for (3.1.4') as for (3.1.3): in the nonhydrostatic domain, where vertical velocity becomes essential for dynamics, the second term in (3.1.4') is small in comparison with the first one, and (3.1.4') reduces to (3.1.1). In the following, we will keep to the approximation (3.1.1).

Thus, the dynamic equations of the pseudo-anelastic model are (2.2.1'), (3.1.1), (3.1.2), the continuity equation for anelastic medium

$$\nabla \cdot \mathbf{v} + \frac{\partial \omega}{\partial p} = 0 \ , \tag{3.1.5}$$

the horizontal momentum equation (with diabatic forcing included)

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\ g\nabla z\ -\ \mathbf{f} \times \mathbf{v}\ + \mathbf{A}_{\mathbf{v}}, \tag{3.1.6}$$

and the temperature equation (with diabatic forcing included)

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\varkappa T\omega}{p} + A_T. \tag{3.1.7}$$

Due to maintenance of (approximate) vertical momentum equation (3.1.2), this model is nonhydrostatic. At the same time, the anelastic approximation (3.1.5) filters internally propagating sound waves, making the model more stable while also supporting larger time step at numerical integration. Physically, equation (3.1.5) is valid for processes for which characteristic time-scale $\tau >> \tau_L = L/c_a$, where L is the characteristic spatial scale of the process and $c_a \approx 335$ m/s is the speed of sound. This condition is valid almost at all spatial scales, and approximation (3.1.5), overall accepted in the hydrostatic dynamics, is actually not less accurate in the nonhydrostatic mesoscale domain.

3.2 The final form of pseudo-anelastic equations

We eliminate w^s and n from further treatment, substituting w^s in (3.1.2) with the help of (3.1.1) and n with the help of (2.2.1'). In addition, nonhydrostatic geopotential $\Phi = gz$ is used instead of isobaric height z, and its thermic and baric components φ and ϕ are separated:

$$\Phi \equiv gz = \varphi + \phi \ , \tag{3.2.1a}$$

$$\varphi \equiv gz^t = gh(x,y) + R\int_p^{p_0} \frac{T}{p'}dp' \ , \tag{3.2.1b}$$

$$\phi \equiv gz^b = \Phi - \varphi \ . \tag{3.2.1c}$$

Such separation is not obligatory, but it permits for explicit representation of baric and thermic counterparts in the dynamic equations, which is advantageous for nonhydrostatic modification of hydrostatic numerical schemes.

The equations then become

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{p^2}{H^2}\frac{\partial\phi}{\partial p} + A_\omega \ , \tag{3.2.2}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla(\varphi + \phi) - \mathbf{f}\times\mathbf{v} + \mathbf{A}_\mathbf{v}, \tag{3.2.3}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\varkappa T\omega}{p} + A_T, \tag{3.2.4}$$

$$\nabla\cdot\mathbf{v} + \frac{\partial\omega}{\partial p} = 0 \ , \tag{3.2.5}$$

where

$$A_\omega = -\frac{p}{H}A_w + \omega\left(\frac{c_v\omega}{c_p p} - \frac{A_T}{T} - \frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\right) \ . \tag{3.2.6}$$

To emphasize that the gas constant R may have significant density effect in the moist air, we have maintained the potential time-dependence of R in this formula, though in the dry air $dR/dt = 0$.

Now we have to introduce the geopotential equation. System (3.2.2) – (3.2.5) represents a closed set of five equations for determination of five unknown fields ω, u, v, T and ϕ. The only diagnostical field here is the baric geopotential ϕ and the only diagnostical equation in this set, which can be employed for its determination, is anelastic condition (3.2.5). Thus, the role of ϕ is to keep the motion non-divergent in time. The explicit equation for ϕ follows after applying the time derivative to (3.2.5):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\nabla\cdot\mathbf{v}+\frac{\partial\omega}{\partial p}\right)=0\ ,$$

or

$$\nabla\cdot\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}+\frac{\partial}{\partial p}\frac{\mathrm{d}\omega}{\mathrm{d}t}-\sum_{i,j=1}^{3}\frac{\partial v^i}{\partial x^j}\frac{\partial v^j}{\partial x^i}=0\ ,$$

$(x^i = x, x^2 = y, x^3 = p,\ v^1 = u, v^2 = v, v^3 = \omega)$. Elimination of time derivatives with the help of momentum equations (3.2.2) - (3.2.3) yields the Poisson equation for ϕ

$$\mathcal{L}\phi\equiv\nabla^2\phi+\frac{\partial}{\partial p}\left(\frac{p^2}{H^2}\frac{\partial\phi}{\partial p}\right)=A^v\ , \tag{3.2.7a}$$

where the volume-distributed source function

$$A^v=-\sum_{i,j=1}^{3}\frac{\partial v^i}{\partial x^j}\frac{\partial v^j}{\partial x^i}-\nabla\cdot(\nabla\varphi+f\mathbf{k}\times\mathbf{v})\ \ +\nabla\cdot\mathbf{A_v}+\frac{\partial A_\omega}{\partial p}\ . \tag{3.2.7b}$$

describes the internal sources for ϕ in the moving atmosphere, determined by different forces trying to change the p-coordinate volume of air particles. The change is prevented by the reaction of the field ϕ. The first sum in (3.2.7b) represents inertial effects. Individual air masses move with different velocities and collide permanently, trying to change the volume of each other. The ϕ, generated by this term as solution of the elliptic equation (3.2.7a), eliminates such volume change effects. Analogically, the second term in A_ϕ is caused by the two-dimensional divergence of ageostrophic acceleration, and the third – by three-dimensional divergence of diabatic forcing.

Equation (3.2.7) is valid in the internal points of the domain. When considering it in the closed domain with boundary points included, the singular sources A^b, located on the boundary surface should be included, which would describe the effect of boundary conditions, induced by the surrounding environment

$$\mathcal{L}\phi = A^v + A^b \; . \tag{3.2.7$'$}$$

Inclusion of boundary sources is essential, when the elliptic equation is solved using the orthogonal basis. Still, specification of A^b requires the preliminary specification of boundary conditions for ϕ. We will return to this problem later.

After the elliptic equation for ϕ is derived, one equation in set (3.2.2) – (3.2.7) will be superfluous and can be left out. We will omit the vertical momentum equation (3.2.2). Then the model to be integrated is represented via equations (3.2.3) – (3.2.7). This is the final form, which will be the basic system for further numerical scheme development. Prognostic equations in this system are the same as in hydrostatic model, but a new scalar field, the baric geopotential ϕ in the horizontal momentum equation (3.2.3), diagnosed from the elliptic equation (3.2.7), is added.

3.3 Geometry of the domain and boundary conditions

The pseudo-anelastic approximation, introduced in sections (3.1) -(3.2) maintains the quality of pressure-space. The vertical coordinate p represents still complete pressure, ω is the true pressure change rate of the individual air particle, and diagnosis of hydrostatic and nonhydrostatic pressure components can be performed using formulae (2.1.3c) and (2.2.4). The difference from exact dynamics is only in the way of evaluation of the density distribution n, required for integral (2.2.4). In exact dynamics it was prognosed from the equation (2.2.3), whereas in the pseudo-anelastic model n is evaluated from z distribution via (2.2.1').

The maintenance of the essence of pressure and pressure velocity involves also maintenance of the geometric domain (2.4.1) and equation of the lower boundary (2.4.2). At the upper boundary we assume condition (2.4.3) for ω. Integration of the continuity equation (3.2.5) yields then a diagnostic expression for ω, which is in common with the hydrostatic model

$$\omega = -\int_0^p \nabla \cdot \mathbf{v} dp' \; . \tag{3.3.1}$$

The combination of this relationship at the level p_0 with (2.4.2) yields the tendency equation for surface pressure

$$\frac{\partial p_0}{\partial t} + \nabla \cdot \int_0^{p_0} \mathbf{v} dp \;=\; 0 \; . \tag{3.3.2}$$

This equation guarantees maintenance of integral conservation laws (of mass, energy and momentum). By appearance it is close to the hydrostatic surface pressure equation (2.4.6) of exact dynamics. Still, there is a principal difference in p_0 and p_0^s in the pseudo-anelastic model, too, and close appearance of (3.3.2) to (2.4.6) should not be misinterpreted as coincidence of p_0 and p_0^s. Pressure p_0 is a prognostic field, which is prognosed from the lower boundary equation (2.4.2) or from the equivalent equation (3.3.2). The hydrostatic surface pressure p_0^s in the pseudo-anelastic model is a diagnostical quantity, for which equation (2.4.6) does not hold anymore and which can be evaluated from (2.2.4) only.

Boundary conditions for the elliptic equation.

The **lower boundary condition** (2.4.4) is not affected by the pseudo-anelastic approximation. For full geopotential, $\Phi = gz$, (2.4.4) gives $\Phi|_{p_0} \;=\; gh$. Thus,

$$\phi|_{p_0} = 0 \; . \tag{3.3.3}$$

The general **lateral boundary condition** is the Neumann condition

$$\left(\frac{\partial \phi}{\partial n}\right)_\Gamma = a_\Gamma \; , \tag{3.3.4}$$

where a_Γ is a given function, which specifies the normal gradient of baric geopotential on the lateral boundary surface Γ. The general idea is, that when the horizontal momentum equation (3.2.3) is presented in the Eulerian form

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{F} - \nabla \phi \; ,$$

where $\mathbf{F}$ represents the "hydrostatic" tendency, then the normal gradient of ϕ must compensate the difference between the normal components of the true and hydrostatic tendencies on the boundary. This gives

$$a_\Gamma = \mathbf{n} \cdot \left(\mathbf{F}_\Gamma - \frac{\partial \mathbf{v_\Gamma}}{\partial t}\right) \; , \tag{3.3.5}$$

where the subscript Γ points, that both $\mathbf{F}$ and $\mathbf{v}$ are externally driven on the boundary. In practice, a_Γ depends much on the applied boundary model. In the present nonhydrostatic approach the lateral relaxation mechanism of the hydrostatic HIRLAM is applied, and the choice of a_Γ for this particular model is discussed later. Until that, a_Γ is treated as a given function.

A fixed **upper boundary condition** for ϕ as a limit $\phi \rightarrow \phi_0$ has no sense due to indefinite nature of the right hand term in (3.2.2) at $p = 0$. Instead, we will apply *the integrability condition*, which also determines behavior of ϕ at $p = 0$ uniquely (for a given lower boundary value):

$$\int_0^{p_0} |\phi| dp < \infty, \quad \text{if} \quad \int_0^{p_0} |A| dp < \infty. \tag{3.3.6}$$

For an explanation, what this condition means, let us consider equation (3.2.7) for constant surface pressure $p_0 = const$ and isothermal atmosphere $H = const$. The application of Fourier transformation in x, y transforms this equation to an ordinary second order differential equation for the Fourier coefficient $\tilde{\phi}(p)$ of the baric potential at wavenumber k

$$\frac{\mathrm{d}}{\mathrm{d}p} p^2 \frac{\mathrm{d}}{\mathrm{d}p} \tilde{\phi} - H^2 k^2 \tilde{\phi} = H^2 \tilde{A} \ ,$$

where $\tilde{A}(p)$ is the Fourier coefficient of source $A(x, y, p)$ at wavenumber k. Two independent solutions of the homogeneous equation are the regular and irregular solutions

$$\tilde{\phi}^{re}(p) = \left(\frac{p}{p_0}\right)^{\mu_k - 1/2} \ , \quad \tilde{\phi}^{ir}(p) = \left(\frac{p}{p_0}\right)^{-\mu_k - 1/2} \ ,$$

where

$$\mu_k = \sqrt{1/4 + H^2 k^2} \ .$$

The regular solution satisfies the condition (3.3.6). The second, irregular solution is unbounded at $p \rightarrow 0$ and does not satisfy (3.3.6). Thus, the irregular solution must be left aside.

When the orthogonal basis is applied in the solution of the elliptic equation, condition (3.3.6) implies lack of singular sources at the upper boundary.

Boundary sources.

Specification of the boundary conditions in the form (3.3.3) - (3.3.6) also makes it possible to fix the structure of the surface source A^b in (3.2.7') as

$$A^b(\mathbf{x}, p, t) = \gamma(\mathbf{x}, t)\delta(p, p_0) + f(\mathbf{x}_\Gamma, p, t)\delta(\mathbf{x}, \mathbf{x}_\Gamma) \ . \tag{3.3.7}$$

Here γ and f are the amplitudes of singular boundary sources, which have to be specified from (3.3.3), and (3.3.4), respectively. At first, the general solution of the elliptic equation is found for the optional distribution of amplitudes γ and f^Γ , and then, γ and f^Γ are determined from the boundary conditions (3.3.3) - (3.3.4). Details of the f^Γ and γ specification will be discussed later, in Chapter 5, when the discrete model is introduced and orthogonal bases are applied for solution of the elliptic equation.

3.4 Surface pressure adjustment

The application of the described pseudo-anelastic model with lower boundary, evolving in accordance with the equation (3.3.2), would yield dynamics with the external mode sound waves included. However, the aim of this non-hydrostatic approach is to develop a model with eliminated external waves and adjusted surface pressure. This would represent a truly anelastic model with completely filtered acoustics. To achieve such a model, it is necessary to modify both the lower boundary p_0 and the lower boundary conditions.

Making use of small amplitude of the relative surface pressure fluctuation $p_0'/\overline{p}_0$, we will consider motion in a given (pre-specified) geometrical domain

$$0 \ < \ p \ < \overline{p}_0(x, y, t) \ , \tag{3.4.1}$$

and linearize quantities and relationships, which depend explicitly on the actual surface pressure, with respect to p_0'. The background field $\overline{p}_0(x, y, t)$ approximates the real surface pressure. However, it is a given field (externally driven as (2.5.7b), or fixed in time like (2.5.7a)), and it is not influenced by actual dynamics of the atmosphere. Linearization affects the kinematic condition (2.4.2) and the expression for geopotential Φ. The linearized form of (2.4.2) is

$$\frac{\partial p_0'}{\partial t} + \frac{\partial \overline{p}_0}{\partial t} + \mathbf{v}|_{\overline{p}_0} \cdot \nabla \overline{p}_0 = \omega|_{\overline{p}_0} \ , \tag{3.4.2}$$

from which the linearized form of vertically integrated mass balance follows with the help of (3.3.1)

$$\frac{\partial p_0'}{\partial t} + \frac{\partial \overline{p}_0}{\partial t} + \nabla \cdot \int_0^{\overline{p}_0} \mathbf{v} dp \;=\; 0 \;. \tag{3.4.3}$$

Surface pressure adjustment means, that the pressure fluctuation tendency term is very small in these equations in comparison with other terms, and approximately

$$\omega|_{p_0} = \;\frac{\mathrm{d}\overline{p}_0}{\mathrm{d}t} \;= \frac{\partial \overline{p}_0}{\partial t} + \mathbf{v}|_{\overline{p}_0} \cdot \nabla \overline{p}_0 \;, \tag{3.4.2$'$}$$

$$\frac{\partial \overline{p}_0}{\partial t} \;+ \nabla \cdot \int_0^{\overline{p}_0} \mathbf{v} dp \;=\; 0 \;. \tag{3.4.3$'$}$$

The geopotential dependence on p_0' becomes evident, splitting Φ into thermic and baric components with respect to $\overline{p}_0$

$$\Phi = \varphi + \phi \;, \tag{3.4.4a}$$

$$\varphi = gz^t = gh(x,y) + R \int_p^{\overline{p}_0} \frac{T}{p'} dp' \;, \tag{3.4.4b}$$

$$\phi \equiv gz^b = \phi^n + \int_{\overline{p}_0}^{p_0} \frac{RT}{p} dp \;\approx \phi^n + \left(\frac{RT}{p} \right)_{\overline{p}_0} p_0'. \tag{3.4.4c}$$

To investigate the adjustment process, we apply time derivative to (3.4.3)

$$\frac{\partial^2 p_0'}{\partial t^2} + \frac{\partial^2 \overline{p}_0}{\partial t^2} + \nabla \cdot \int_0^{\overline{p}_0} \frac{\partial \mathbf{v}}{\partial t} dp \;=\; 0 \;. \tag{3.4.5}$$

Using the horizontal momentum equation (3.2.3) and the results from (3.4.4), equation (3.4.5) is transformed to

$$\frac{\partial^2 p_0'}{\partial t^2} - \nabla \cdot \left[\overline{p}_0 \nabla \left(\frac{c^2 p_0'}{\overline{p}_0} \right) \right] = A_p \;, \tag{3.4.6a}$$

$$A_p = \nabla \cdot \int_0^{\overline{p}_0} [\mathbf{v} \cdot \nabla \mathbf{v} + \omega \frac{\partial \mathbf{v}}{\partial p} + \nabla(\varphi + \phi^n) + f\mathbf{k} \times \mathbf{v} - \mathbf{A}_\mathbf{v}] dp - \frac{\partial^2 \overline{p}_0}{\partial t^2} \;, \tag{3.4.6b}$$

where $c \;=\; \sqrt{(RT)_{\overline{p}_0}} \approx 280$ m/s is the isochoric sound speed. This is the nonhomogeneous wave equation for surface pressure fluctuation. A special case of equation (3.4.6a) is a model with uniform bottom, $\overline{p}_0 = \text{const.}$, when (3.4.6a) simplifies to

$$\frac{\partial^2 p_0'}{\partial t^2} - \nabla^2(c^2 p_0') = A_p \ .$$

Dynamics of surface pressure can be treated as adjusted, if the first term in (3.4.6a) is much less than the second one:

$$\left| \frac{\partial^2 p_0'}{\partial t^2} \right| << \left| \nabla \cdot \left[\overline{p}_0 \nabla \left(\frac{c^2 p_0'}{\overline{p}_0} \right) \right] \right| \tag{3.4.7}$$

and when (3.4.6a) simplifies to

$$-\nabla \cdot \left[\overline{p}_0 \nabla \left(\frac{c^2 p_0'}{\overline{p}_0} \right) \right] = A_p. \tag{3.4.8}$$

Let L and τ be the characteristic spatial and temporal scales of motion under consideration. Then standard scale analysis of (3.4.7) leads to the condition

$$\tau \ >> \ \tau_L \equiv L/c \ . \tag{3.4.9}$$

In limited area models, L is of the the same order as the horizontal extent of the domain of integration. In such a situation, the condition (3.4.8) means that the characteristic time-scale of the process must be larger than the time interval, required for sound waves (propagating with speed c) to travel out of the domain. The condition (3.4.9) is in practice valid at all spatial scales L, as for slow processes $\tau \ \sim \ L/v$, where $v \sim 10$ m/s. Note that adjustment is achieved formally by increasing the sound speed to infinity ($c \rightarrow \ \infty$) in (3.4.7).

In the adjusted model, the surface pressure consists of the mean steady component and small adjusted contribution

$$p_0 = \overline{p}_0 + p_0' \ . \tag{3.4.10}$$

The fluctuation p_0' can be evaluated from the equation (3.4.8), if the right hand term is known. This is the case of hydrostatic dynamics. In nonhydrostatic region, the right hand term A_p includes nonhydrostatic geopotential ϕ^n which is not known separately from baric geopotential ϕ. Therefore it is necessary to move ϕ^n in (3.4.8) to the left side and, with the help of (3.4.4c), present (3.4.8) as an integral condition for ϕ

$$\nabla \cdot \int_0^{\overline{p}_0} \nabla\phi dp = -\nabla \cdot \int_0^{\overline{p}_0} \left(\mathbf{v}\cdot\nabla\mathbf{v} + \omega\frac{\partial \mathbf{v}}{\partial p} + \nabla\varphi + f\mathbf{k}\times\mathbf{v} - \mathbf{A_v} \right) dp + \frac{\partial^2 \overline{p}_0}{\partial t^2} \ , \tag{3.4.11}$$

In the adjusted case this relationship must be used for specification of the boundary source amplitude γ in (3.3.7). In physical aspect, (3.4.11) controls the temporal maintenance of the vertically integrated mass balance (3.4.3'). Namely, it validates the condition (3.4.3') for all times, if that condition is satisfied at the initial moment.

As it became evident, in the adjusted model the surface pressure fluctuation is represented implicitly in the body of the baric geopotential (3.4.4c). However, it is not completely invisible and can be diagnosed from ϕ using an extrapolation

$$\Phi|_{p_0} \approx \Phi|_{\overline{p}_0} + \left(\frac{\partial \Phi}{\partial p}\right)_{\overline{p}_0} p_0' = gh \ .$$

With the help of (3.4.4) we obtain

$$\Phi|_{\overline{p}_0} = \varphi_{\overline{p}_0} + \phi_{\overline{p}_0} = gh + \phi_{\overline{p}_0}, \quad \left(\frac{\partial \Phi}{\partial p}\right)_{\overline{p}_0} = \left(\frac{\partial \phi}{\partial p} + \frac{\partial \varphi}{\partial p}\right)_{\overline{p}_0} = \left(\frac{\partial \phi}{\partial p} - \frac{RT}{p}\right)_{\overline{p}_0} \ ,$$

and the extrapolation formula reduces to

$$\left(\frac{RT}{p} - \frac{\partial \phi}{\partial p}\right)_{\overline{p}_0} p_0' = \phi|_{\overline{p}_0} \ . \tag{3.4.12}$$

Usually, the second term in the first brackets is small in comparison with the first, and approximately

$$\left(\frac{RT}{p}\right)_{\overline{p}_0} p_0' = \phi|_{\overline{p}_0} \ . \tag{3.4.12$'$}$$

The omega-velocity (3.3.1) can be diagnosed in the adjusted case with the help of (3.4.3') as

$$\omega = \frac{\partial \overline{p}_0}{\partial t} + \mathbf{v}|_{\overline{p}_0} \cdot \nabla\overline{p}_0 + \int_p^{\overline{p}_0} \nabla\cdot\mathbf{v} dp' \ , \tag{3.4.13}$$

or in the equivalent form

$$\omega = \frac{\partial \overline{p}_0}{\partial t} + \nabla \cdot \int_p^{\overline{p}_0} \mathbf{v} dp' \ . \qquad (3.4.13')$$

This relationship is used in the nonhydrostatic anelastic model elsewhere. As the condition (3.4.3') arises from (3.4.11) at every $t > 0$ on the constraint that (3.4.3') holds for t=0, the relationship (3.4.13') also provides the upper boundary condition $\omega \ \rightarrow \ 0$ (see (2.4.3)) at every $t > 0$, at the additional constraint that this condition is satisfied at $t = 0$. .

The set of equations (3.2.3) - (3.2.4), (3.2.7), (3.4.4b), (3.4.13), when treated in the domain (3.4.1), represents the anelastic, nonhydrostatic, pressure coordinate model, as it completely lacks any (internal or external) acoustic mode.

3. 5. Anelastic model in pressure coordinates: Summary

The closed set of relationships and equations, needed for integration of the developed anelastic, pressure-coordinate model, is as follows.

The domain of integration is fixed in pressure–coordinates and is presented by (3.4.1).

The prognostic fields of the model are the wind components u, v, and the temperature T. The consequent prognostic equations are the horizontal momentum equation (3.2.3) and temperature equation (3.2.4).

The baric and thermic geopotential components, which together determine the horizontal gradient–force in the momentum equation (3.2.3), are both diagnostic fields. The thermic geopotential φ is diagnosed from (3.4.4b). The baric geopotential ϕ is diagnosed from the elliptic equation (3.2.7'), which is solved in the domain (3.4.1) with boundary conditions (3.3.4), (3.3.6) (with $\overline{p}_0$ instead of p_0 in the role of the upper boundary of integrals), and (3.4.11).

The omega–velocity represents also a diagnostic field, which is diagnosed from (3.4.13) or from (3.4.13').

The actual surface pressure presents an auxiliary field, which can be diagnosed from the relationship (3.4.12).

Comments

The general model, described in Section 2, is a perfect pressure-coordinate equivalent of the general equations of atmospheric dynamics, commonly treated in Cartesian coordinates. However, the filtered anelastic model, derived in Section 3, has not been widely used and has no direct analogues in the family of numerical weather prediction models. In this respect, it is of certain interest to follow possible links of this model with the most familiar approaches in the numerical weather prediction practice.

As it appears, the closest relative to the general pressure–coordinate (*p-coordinate*) model (2.2.1') - (2.3.4) is the general (fully elastic) hydrostatic–pressure–coordinate ($p^s - coordinate$) model (Laprise 1992, 1998). These two models actually represent the same general case, realized in (slightly) different coordinate systems. The transformation of equation from p-coordinate system to the p^s-coordinate system and *vice versa* is straightforward. The corresponding coordinate transformation is represented by the relationship (2.2.4').

However, the further implementation of these equations goes in the Laprise' approach and in the present model in different ways.

In the Laprise' approach, the initial fully elastic equations are taken as a basis for numerical implementation. The fast acoustic and buoyancy modes are handled via semi-implicit reformulation of equations, making use of the Tanguay-Robert-Laprise (1990) hypothesis. That would be an option in the present case, too; for that it would be necessary to formulate equations of motion (2.2.3), (2.3.1) - (2.3.4) in hybrid coordinates, and then present them in the semi-implicit numerical framework. The essence of the semi-implicit approach is, as wellknown, the reduction of the propagation speed ($c \to 0$) of fast acoustic and buoyancy disturbances, retaining the slow advective-convective component of dynamics undistorted, and enhancing this way the numerical stability.

In the present approach, the fast acoustic mode is removed (relaxed, eliminated), before the numerics is introduced, making use of infinite sound speed

($c \rightarrow \infty$) approximation. This is achieved by removing both the internal acoustic mode (using approximation $n \rightarrow 1$ in (2.2.3)), and the external acoustic wave (replacing (3.4.5) by (3.4.11)). It is notable, that the internal wave removal is not sufficient alone (as this is used already in the hydrostatic model without any visible improvement of numerical stability), a decisively new quality is introduced by the complete elimination of transient acoustic waves. The approach is "wave-selective": it removes acoustic waves completely, yet maintains the buoyancy waves unchanged (this was checked and proven to be true, at least, for linear disturbances).

Note that the present anelastic model supports further implementation of the semi-implicit scheme *á la* Tanguay-Robert-Laprise' hypothesis for removal of fast transient buoyancy mode, after which it would be closer to the semi-implicit fully-elastic (Laprise 1998) model.

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