# A novel non-hydrostatic, semi-implicit, semi-Lagrangian scheme for limited-area NWP models

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# Introduction

- Potential of SISL approach

***SISL = Semi Lagrangian, semi-implicit model/approach/method for numerical integration of hydro- and gaso-dynamical Eq.-s***

*Powerful: good accuracy, large stability and significant time steps*

- In atmospheric dynamics, SISL is used in NWP models ~ 10 years **(ECMWF, 1995, Richie et al)**
- New challenges in NH (meso-β and –γ) domains

A novel **nonhydrostatic, SISL, adiabatic core for HIRLAM** has been created at Tartu University.

- new non-hydrostatic core
- novel is the SISL scheme

# NONHYDROSTATIC LAGRANGIAN Eq.-s in PRESSURE COORDINATES

$$\frac{d\omega}{dt} = -\frac{p^2}{H^2}\frac{\partial \phi}{\partial p}$$

$$\frac{d\mathbf{v}}{dt} = -\nabla(\phi + \varphi) - \mathbf{f} \times \mathbf{v}$$

$$c_p \frac{dT}{dt} = RT\frac{\omega}{p}$$

$$\frac{dp_s}{dt} = \omega\big|_{p_s}$$

$$\nabla \cdot \mathbf{v} + \frac{d\omega}{dp} = 0$$

$$\frac{d\varphi}{dp} = -\frac{RT}{p}$$

*Short notation:*

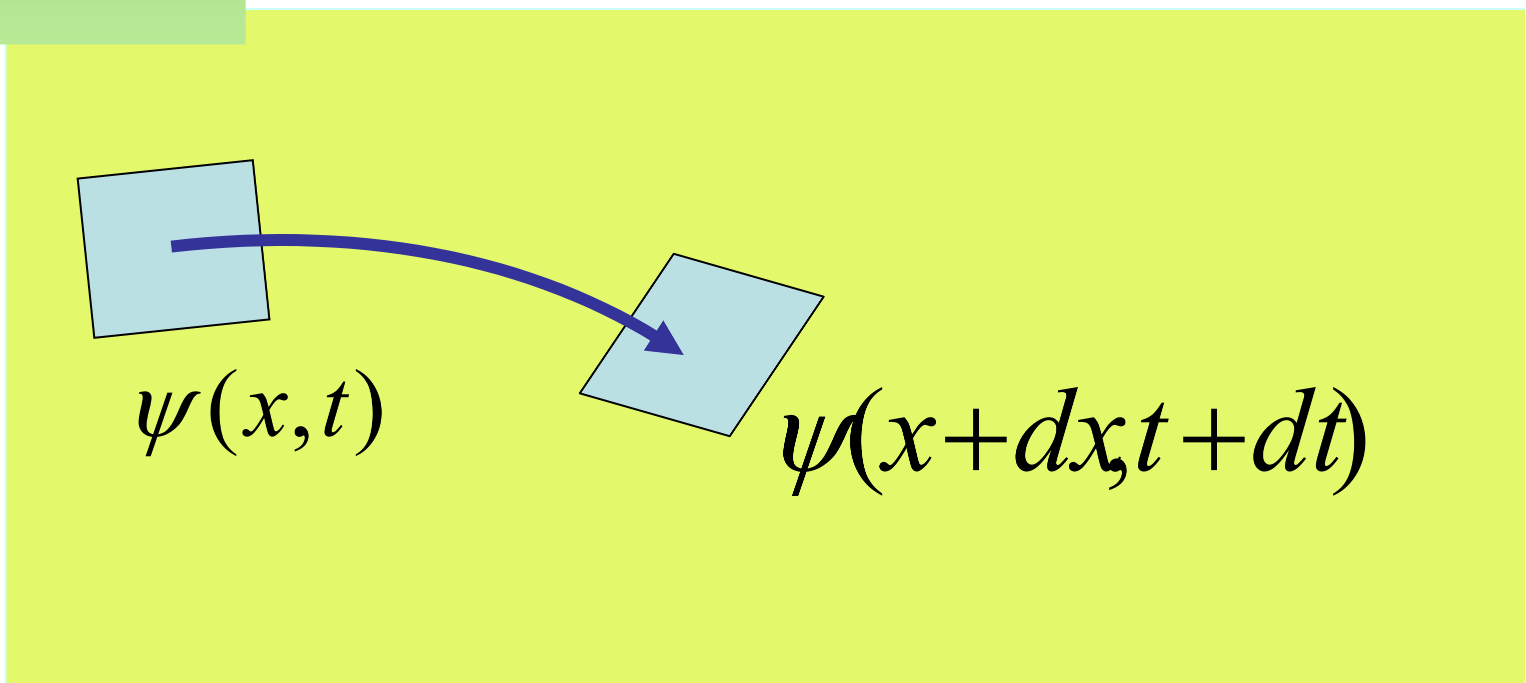
$$\frac{d\psi}{dt} = F(\psi)$$

**v** –wind vector, ***ω*** – omega-velocity, ***H=RT/g*** – scale-height, ***Φ*** – nonhydrostatic geopotential, ***φ*** - hydrostatic geopotential, ***R***, $C_p$ – gas constants, ***p*** – pressure, $p_s$ - surface pressure

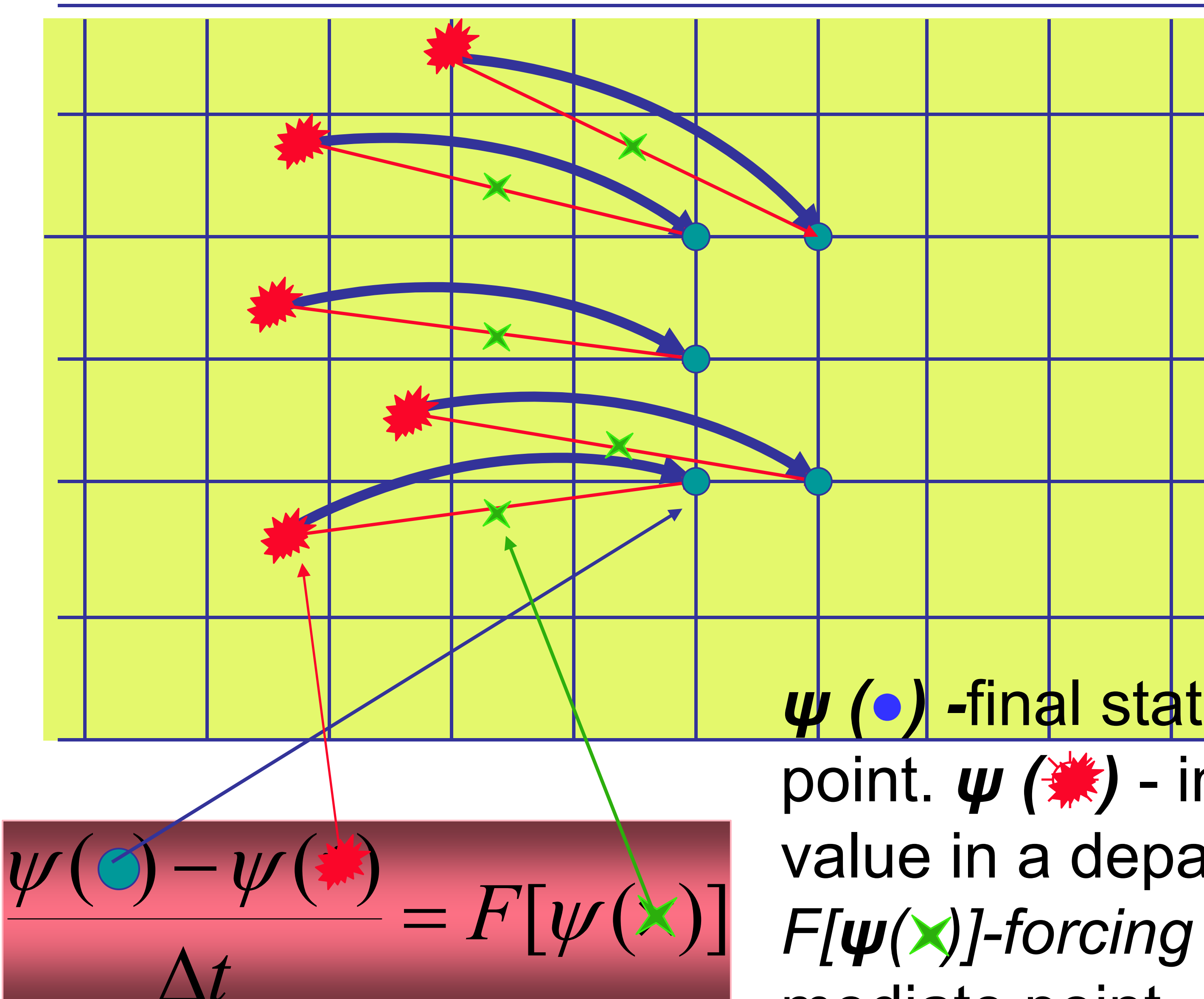
# SISL: how it works

General Lagrangian equation of motion:

$$\frac{d\psi}{dt} = F(\psi)$$



# Semi-Lagrangian scheme (2- and 3-time-level)




***ψ* (●) -**final state in a grid-point. ***ψ* (✸)** - interpolated value in a departure point, *F[**ψ**(✕)]-forcing* in an intermediate point

**Separation of forcing to Linear and nolinear terms…**

$$\psi = \psi^{r} + \psi'$$

$$\frac{\psi(\bullet) - \psi(*)}{\Delta t} = F[\psi(\times)]$$

$$F[\psi(\times)] = L_{\psi^r}\psi'(\times) + N_{\psi^r}[\psi'(\times)]$$

**…which is followed by transfiguration of *L* to an implicit term**

$$(1/2)[L(\bullet) + L(*)] + N(\times)$$

**An important problem is the optimum choice of the reference field(s) $\psi^r$**

**Criterion for optimization: maximizing of the linear term $L$ with respect to the nonlinear term $N$**

$$L_{\psi^r}\psi' \sim \psi',$$

$$N_{\psi^r}(\psi') \sim (\psi')^2$$

**Maximizing of $|L|/|N|$ means minimizing of**

$$|\psi'| = |\psi - \psi^r| \quad !!!$$

***Addional restriction: $\psi^r = \psi^r(p)$***

***(ref. state is horizontally homogeneous).***

**For present NH model, the actual single prime reference field is temperature *$T^r(p)$*.**

**Secondary reference fields are the mean surface pressure, defined via barometric formula**

$$p_s^r(x,y) = p_0 \exp\left(-(g/R)\int_0^{h(x,y)} dz/(T^r)\right)$$

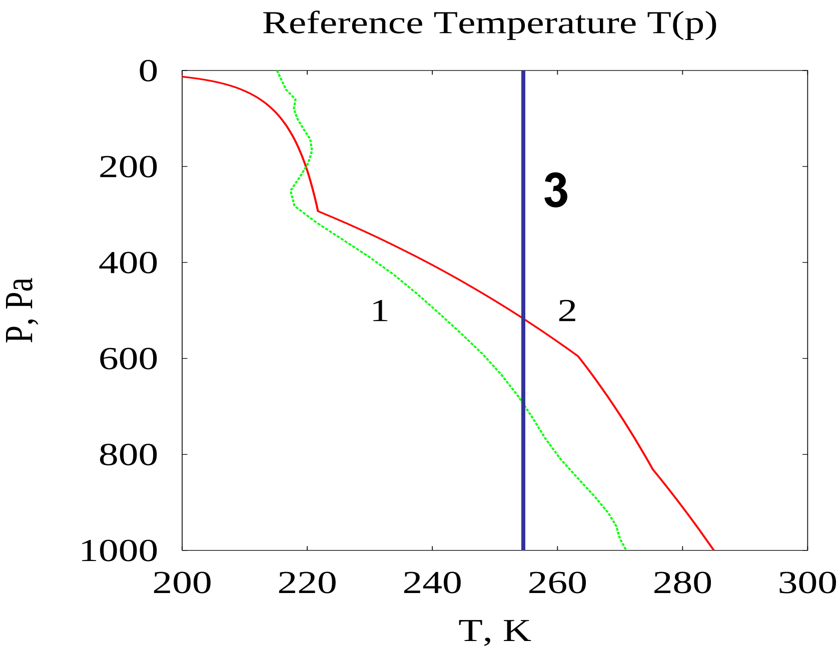
**and the reference hydrostatic geopotential**

$$\varphi^r(x,y,p) = gh(x,y) + R\int_p^{p_s^r(x,y)} T^r(p')dp'/p'$$

In the numerical scheme, the reference temperature is computed as the area-mean over isobaric surface:

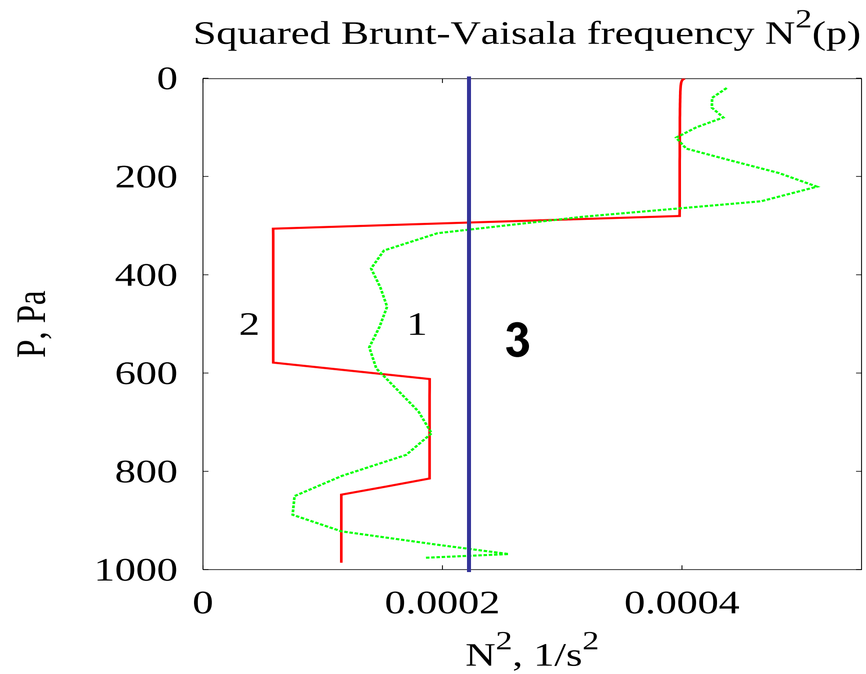
$$T^r(p,t) = \frac{1}{MN}\sum_{mn} T(x_m, y_n, p, t)$$

Thus, $T^r$ is a (slow) function of time.




**Examples of reference temperature...**

**...and corresponding Brunt-Väisälä frequency**




1 - Area-mean temperature over Norway 2001.03.22

2 - Model ***T(p)***, proposed by F. Bouttier for tests and model inter-comparison

3 – Model with constant ***T***

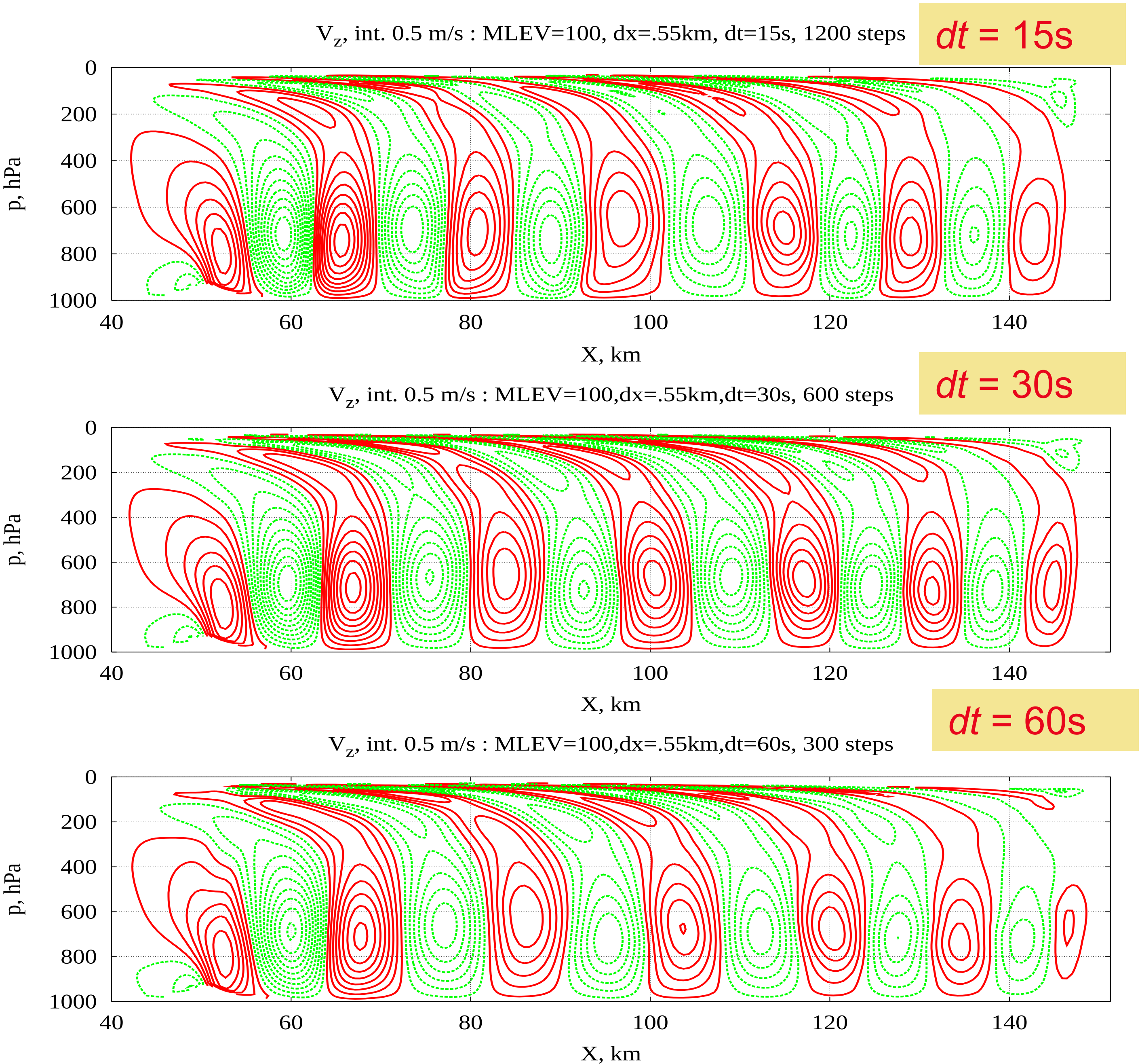
## Numerically the model is performed as the NH extension (adiabatic core) for HIRLAM

- **Hybrid (ECMWF) coordinates, C-grid staggering**
- **Semi-Lagrangian (SL) trajectory calculations**

**For trajectory calculations and interpolating the existing (McDonald & Haugen) routines from HS HIRLAM are employed**

- **Two level time stepping**

# Flow over Agnesi ridge, $a_x$ = 3 km, $h$ = 600 m




$$dt_{cr} = 13s$$

***dx*** = .55 km,

276x100 grid,

100 levels

Integration period: 3 h

Reference state (*U(p)* and *T(p)* ):

Bouttier' profiles

# Time step estimates

There is no strict upper limit of max $\boldsymbol{\Delta t}$ for NH SISL schemes.

In the table, estimations of maximum reliable $\boldsymbol{\Delta t}$ are presented

| Max $\boldsymbol{U}$, [$m/s$] | $\boldsymbol{\Delta x}$, [$km$] | $\Delta t_{Cr}=\Delta x/U$ [$min$] | $\Delta t$ [$min$] | $\Delta t/\Delta t_{Cr}$ | parcel path, [$km$] |
|---|---|---|---|---|---|
| 40 | 5.5 | 2.3 | 4.6 | 2.0 | 11 |
| 55 | 5.5 | 1.67 | 4.0 | 2.4 | 13.2 |
| 40 | 2.2 | 0.92 | 2.8 | 3.0 | 6.6 |
| 42 | 0.55 | 0.22 | 1.0 | 4.6 | 2.5 |

# Real-condition tests

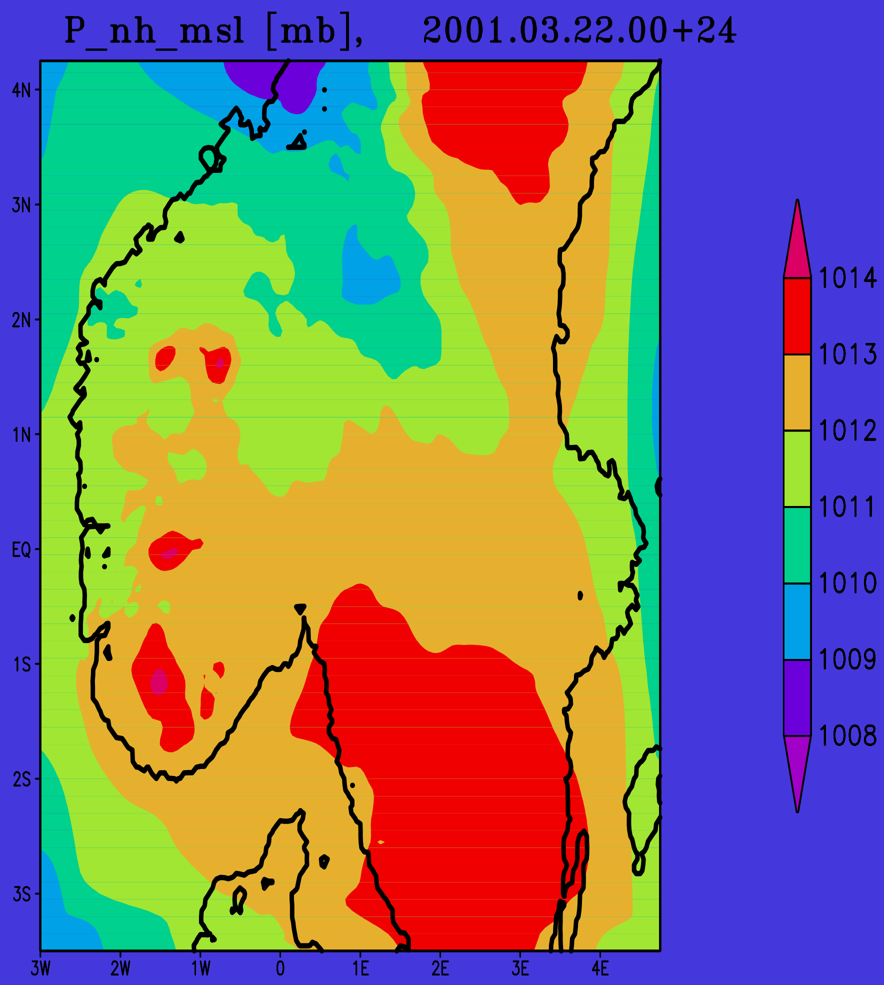
## Norwegian experiment (mountains)

- Resolution 5.5 km
- Grid 156x156, 31 levels
- HIRLAM 5.0.0

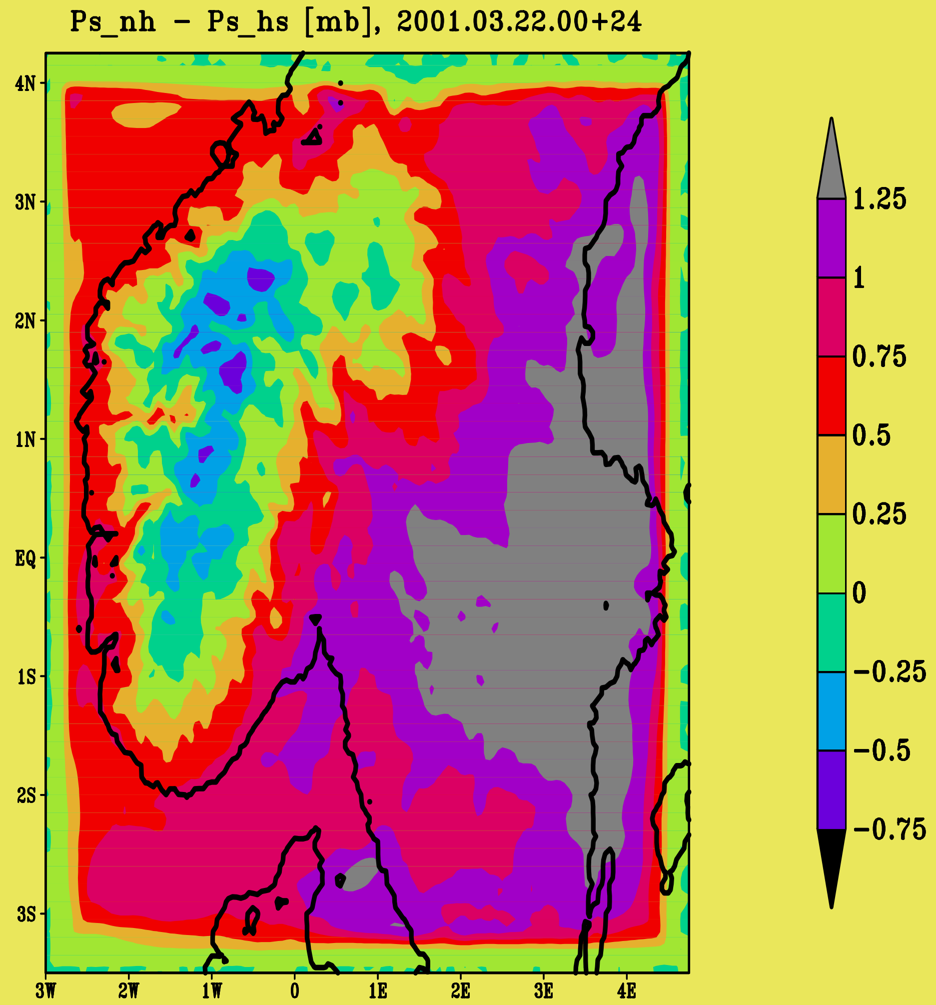
# MSL pressure 24 h forecast. *Resolution 5.5 km, time-step 4 min*

NH model gives ~1 mb larger MSL pressure on plane, ~1 mb lesser at mountain tops

## NH SISL



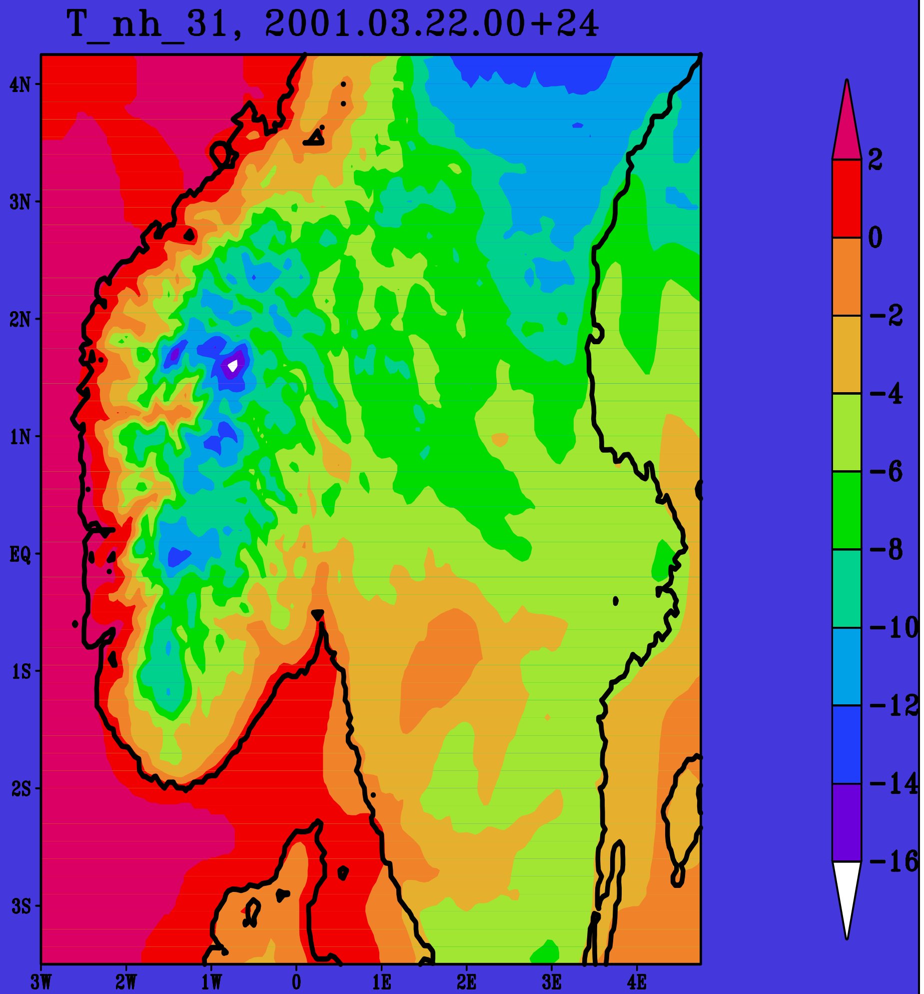

## NH – HS SISL



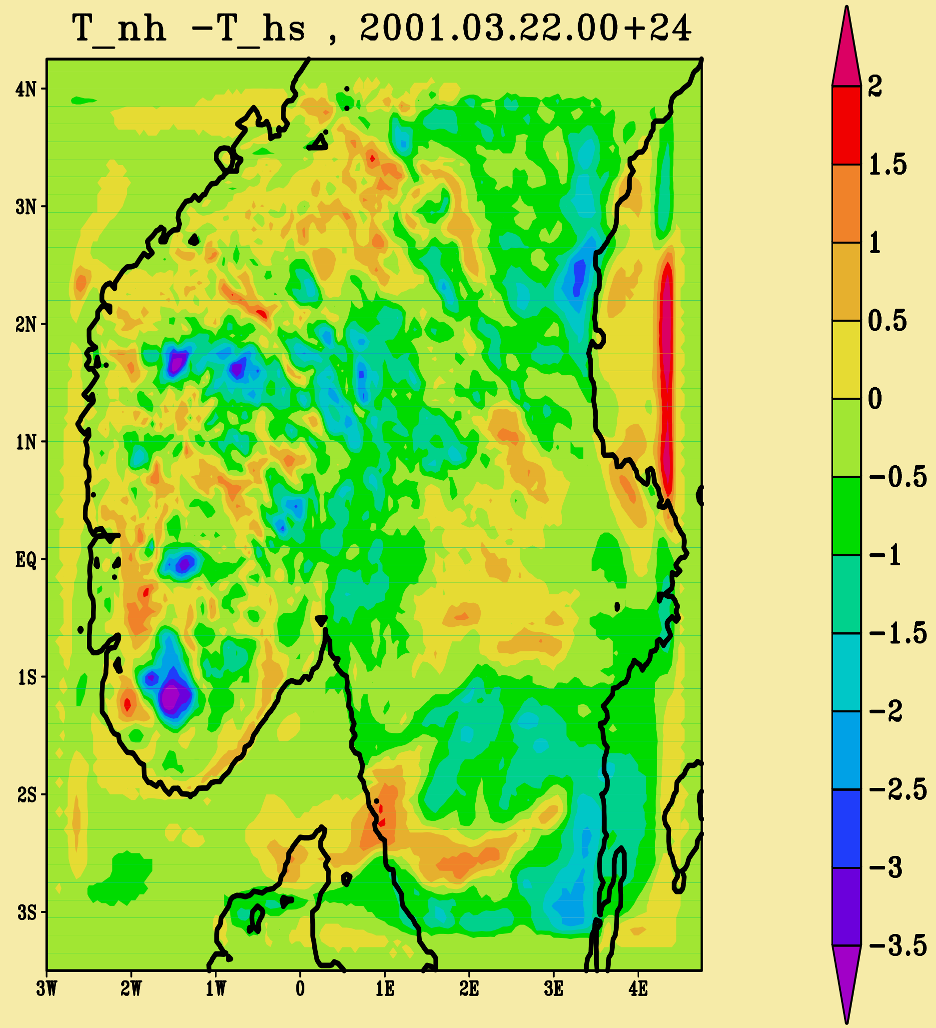
# 10 m temperature, 24 h forecast. *Resolution 5.5 km, time-step 4 min*

Temperature differences are mostly < 0.5 C, except on mountain tops, where ***ΔT***~ -3 C

## NH SISL




## NH - HS SISL

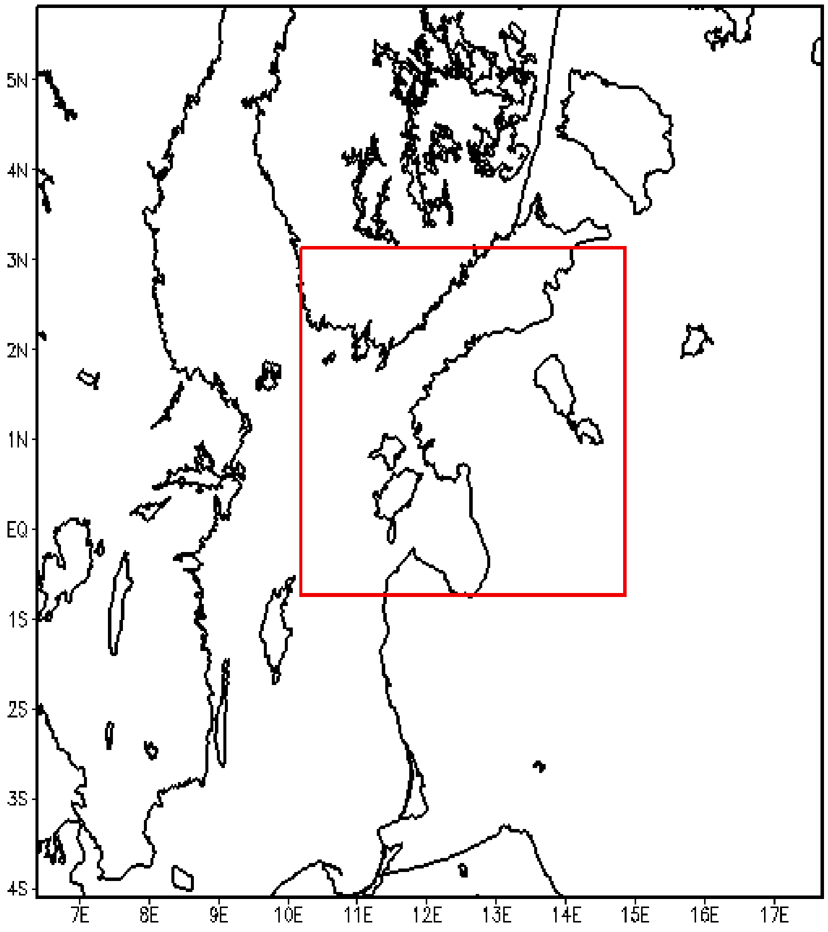


# Applications

**NWP:**

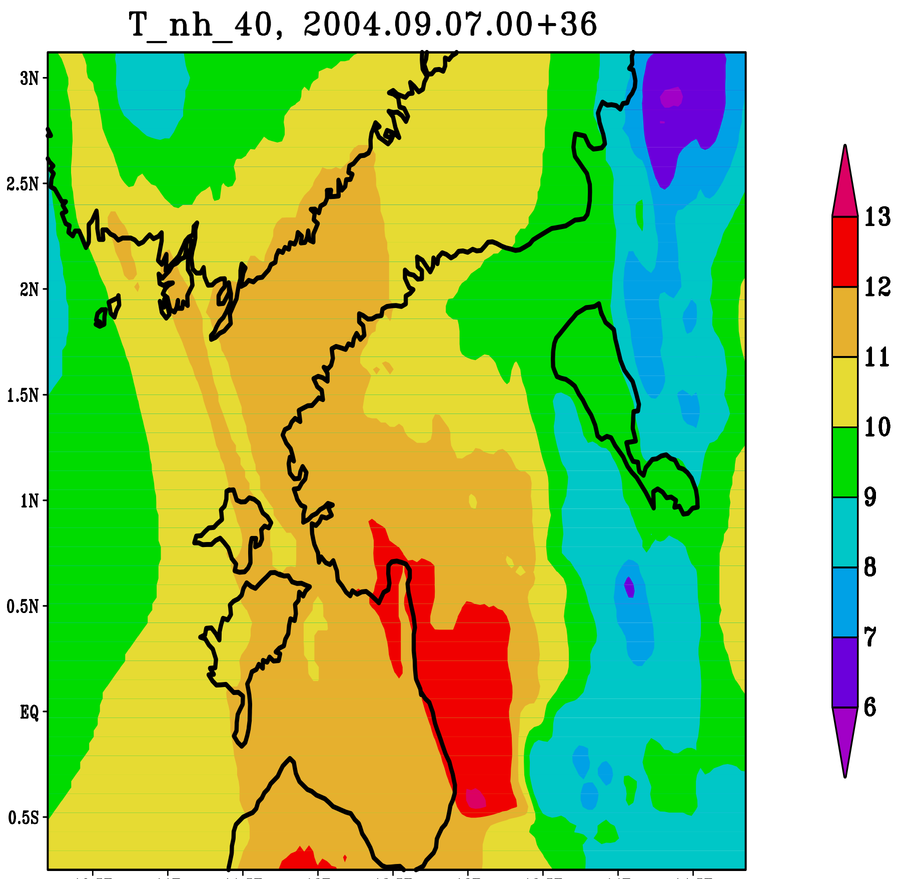
**ETB (Estonian B-area)**

- **resolution 3.3 km,**
- **Grid 186×170×40 (former 104x100x40, area increase 3.3 x )**
- **Δt=150s**
- **Reference HIRLAM 6.1.0**

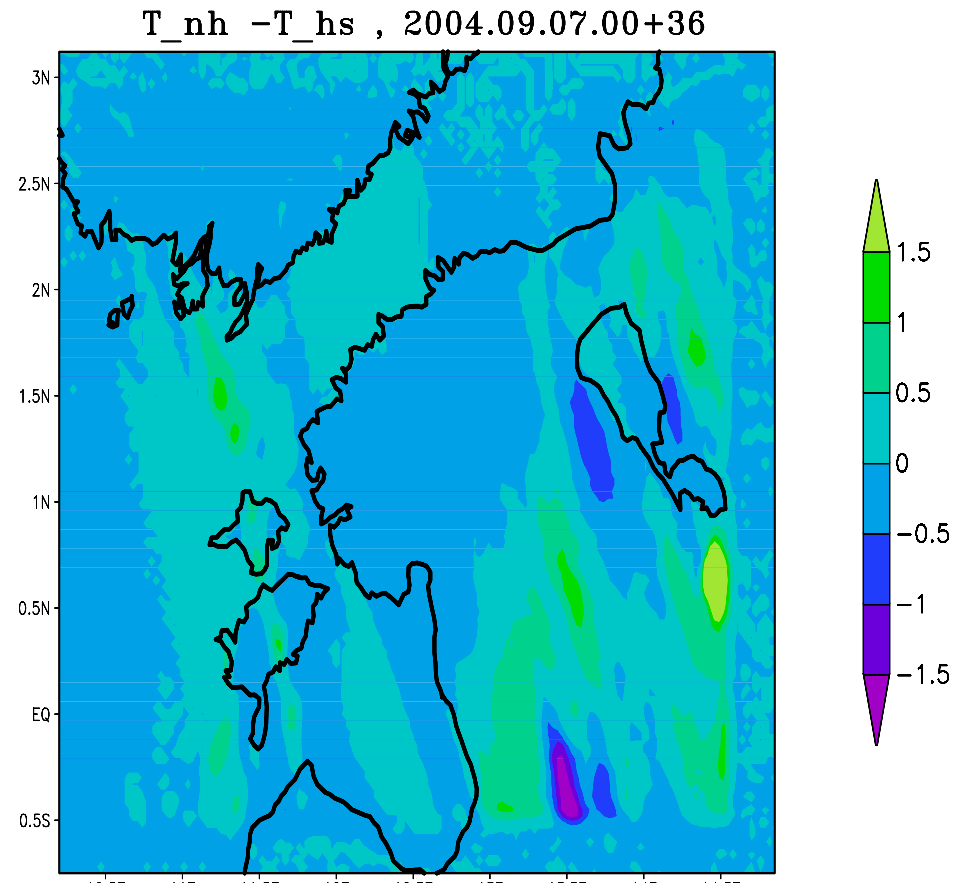


# T_40  36 h forecast within ETB. *Resolution 3.3 km, time-step 1.5 min*

**NH SISL**




**NH – HS SISL**

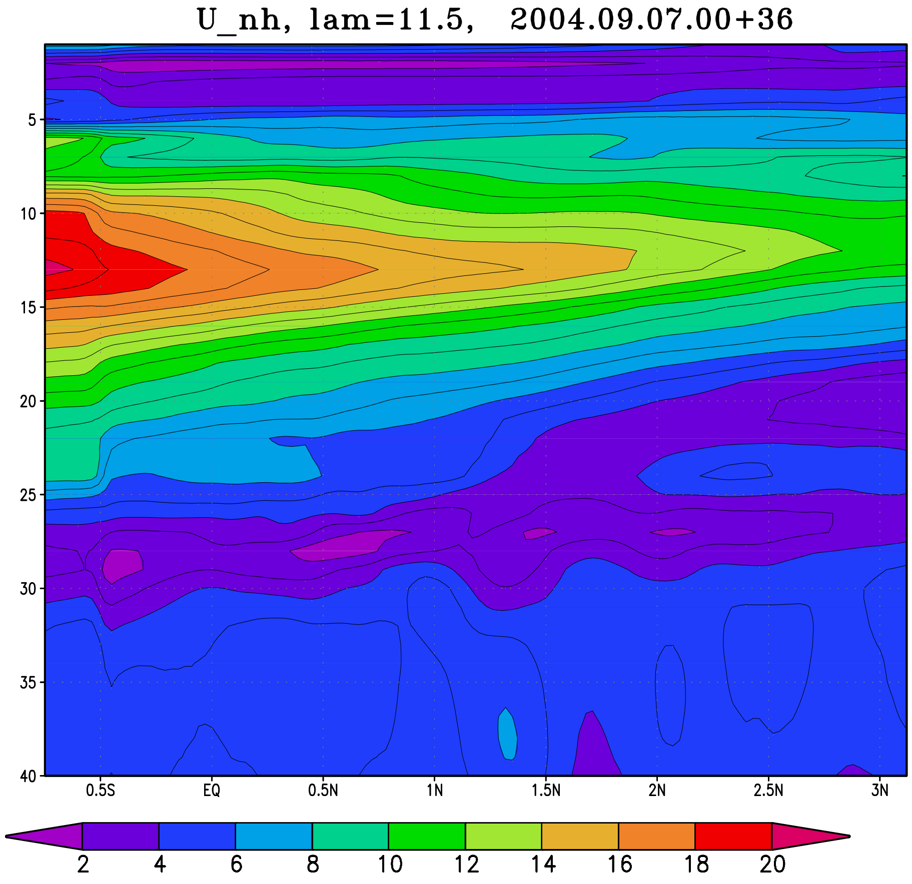


# 36 h forecast within ETB

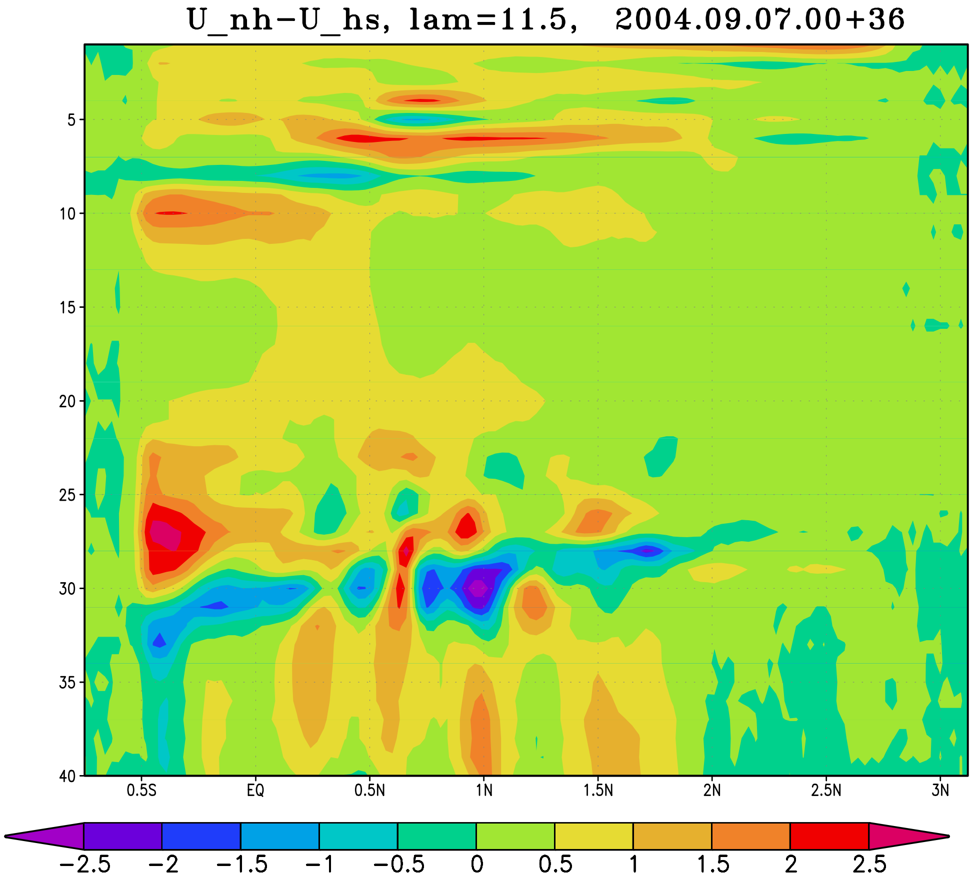
# U-wind vertical cross section

# *Resolution 3.3 km, time-step 1.5 min*

## NH SISL




## NH – HS SISL



# Conclusions

**A new NH SISL scheme with variable (height dependent) reference state is developed:**

- The stability and the time step are increased
- Comparison with theoretical results (mountain flows), as well as with other models (NH Euler, HS SISL) shows that the new scheme is reliable and ready for further applications

**Currently, the NH SISL is implemented as the adiabatic core in ETB** (3.3 km resolution, grid 186x170, 40 levels, physics of HIRLAM 6.1.0) and the testing is activated.

**The new NH SISL will be a suitable tool for model development** (*complex terrain, boundary layer, moist convection*) at very high spatial resolutions (0.5 - 1km, 100 levels and more)