Semi-Lagrangian Approach to 4D-Discrete Linear Equations of Atmospheric Dynamics with Arbitrary Stratification and Orography

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1 Introduction

A method for numerical solution of non-hydrostatic linear equations of atmospheric dynamics for horizontally homogeneous but otherwise arbitrary reference state and arbitrary orography is introduced.

The developed solution is

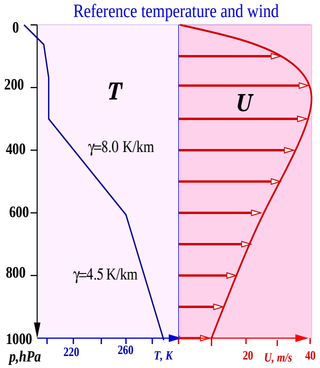
4D-discrete (x,y,z,t),
spectral,
semi-implicit,
semi-Lagrangian
(SISL) scheme

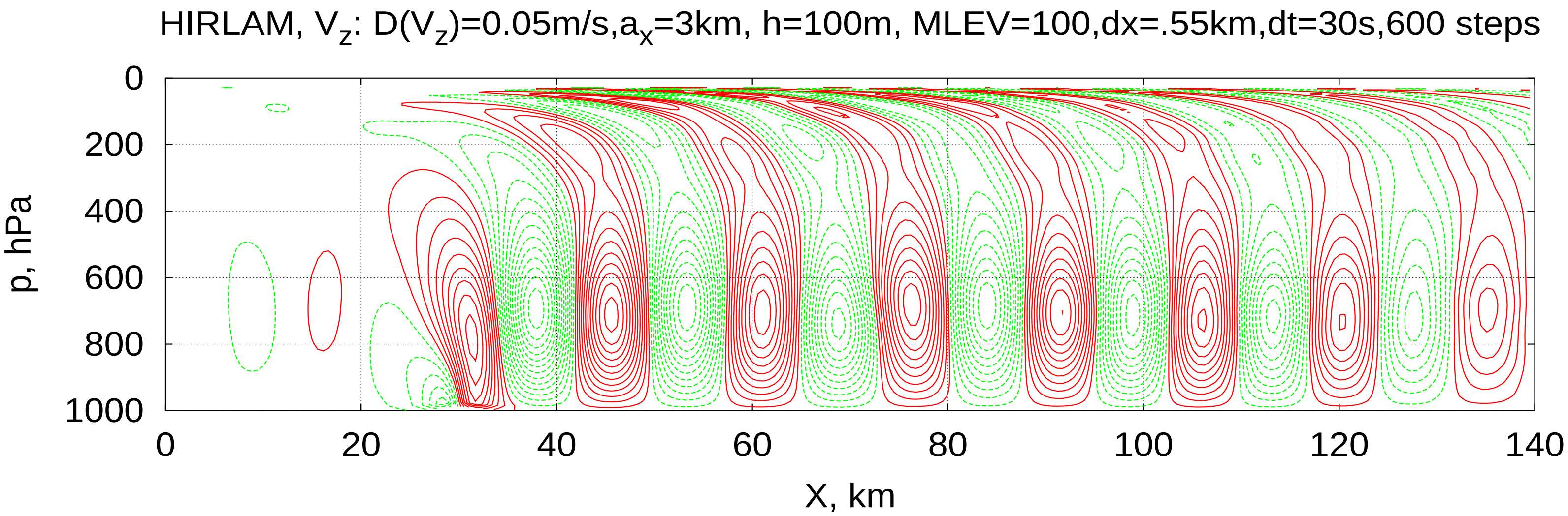
for both stationary and nonstationary cases

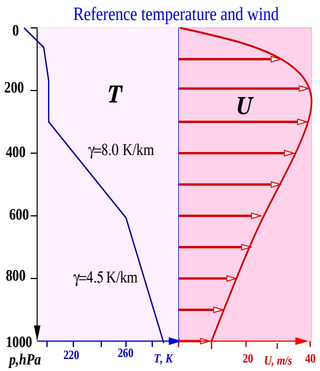
Motivation

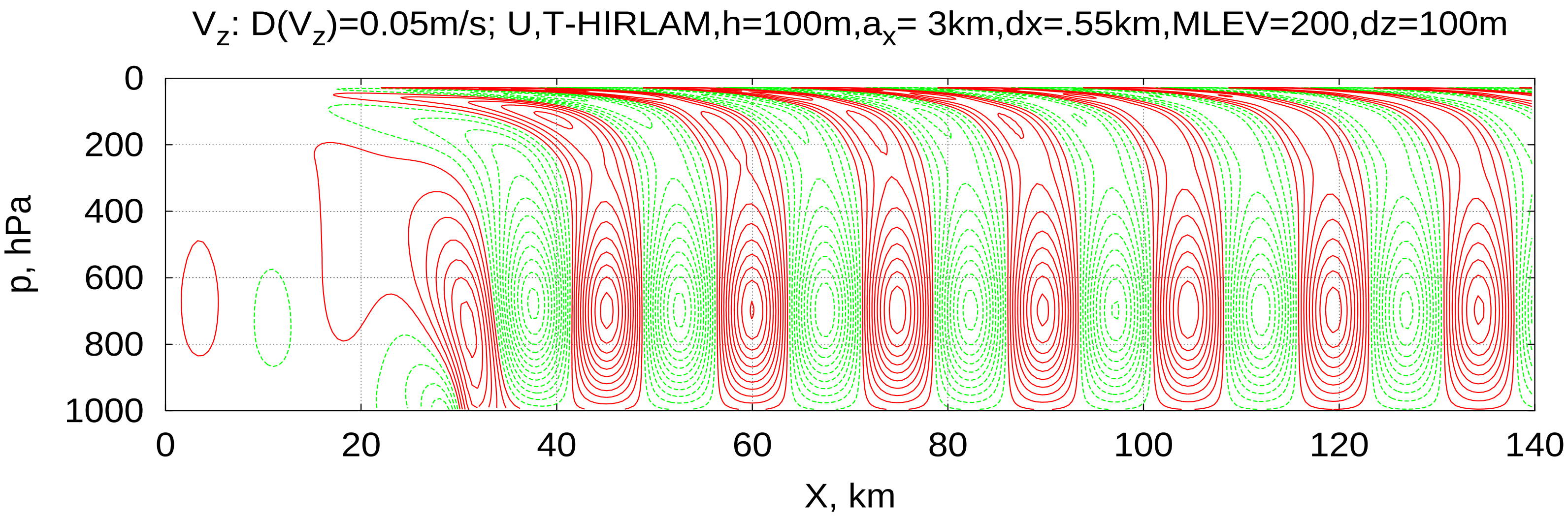
Originally this algorithm was developed for testing and quality check of nonhydrostatic adiabatic kernels of SISL-based NWP models (HIRLAM, in particular)

Testing NH HIRLAM









The actual domain of application is much wider:

Investigation of specific details of orographic flows for complex wind and temperature stratification.

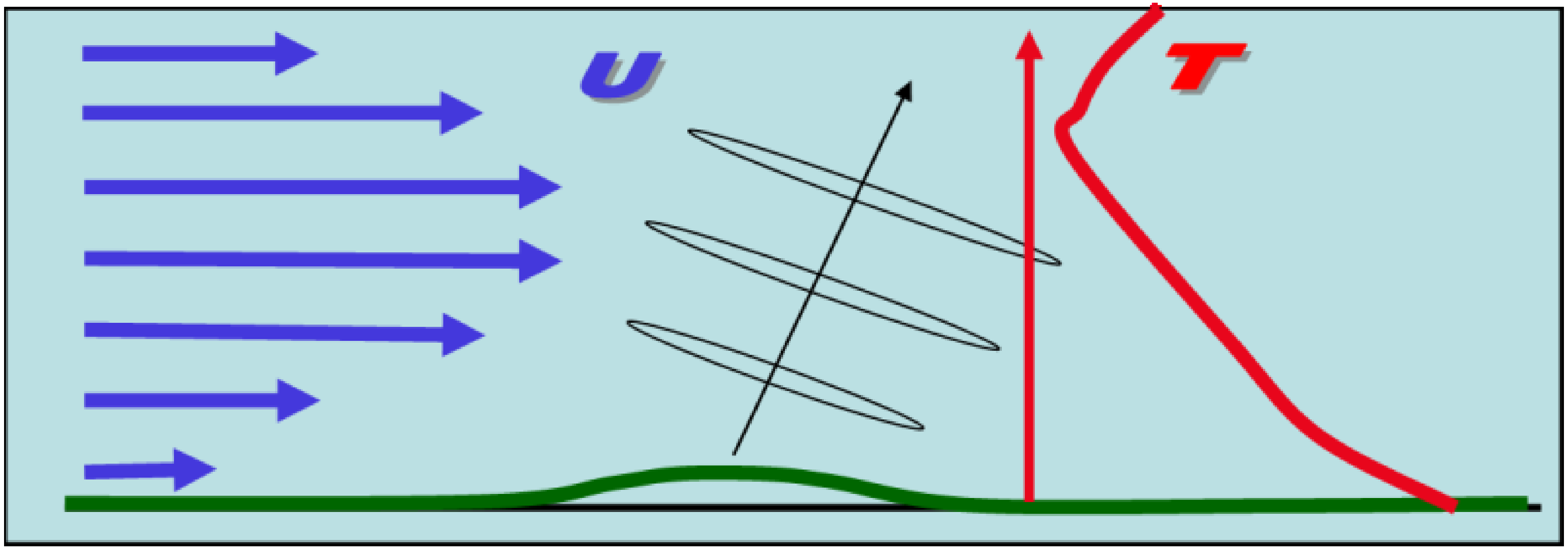
Investigation of non-stationary development and buoyant instability.

Investigation of the impact of discretization to the solution quality.

2 Model description

Initial continuous equations: **Linear, NH pressure-coordinate equations with filtered internal sound-waves**
(Miller-Pearce-White model)

Linearization with respect to $T(p)$, $\mathbf{U}(p)$, $p_s(x,y)$



Discretization:

3D staggering with constant horizontal grid-step $\Delta x = \Delta y$ and variable vertical step Δp_k

Two time level,

semi-implicit,

semi-Lagrangian time scheme

Solution in the form of discrete Fourier series

3D (x,y,t) presentation of discrete solution:

$$\Psi_{ijk}^{n} = \sum_{qrs} \hat{\Psi}_{qrk}^{s} \mathrm{e}^{\mathbf{i}(\eta_q^x i + \eta_r^y j - d_{qrk}^s n)},$$

where

$$\Psi = \{T', u, v, \omega, \varphi\}$$

Discrete spectral equations for spectral amplitudes arrive from which a one-dimensional wave equation follows for ω-velocity.

The wave equation is solved for boundary conditions :

Free-slip condition on the surface

Radiative boundary condition on the top

A special feature of the wave equation:

As normal mode intrinsic frequency ν depends on height, the wave-equation coefficients are functions of both ν and $\Delta\nu/\Delta p$

In stationary case height-dependent ν presents an ordinary thing; In nonstationary case it involves a dispersion equation which is a nonlinear first order differential equation with respect to ν

Special: **Solution of wave equation is designed as a cumulative product of decrease factors**

$$\omega_k = \prod_{j=1}^{k} c_j, \quad |c_j| < 1,$$

which results in an effective numerical algorithm, where solution ω_k **is alwas an exponent function of a complex argument.**

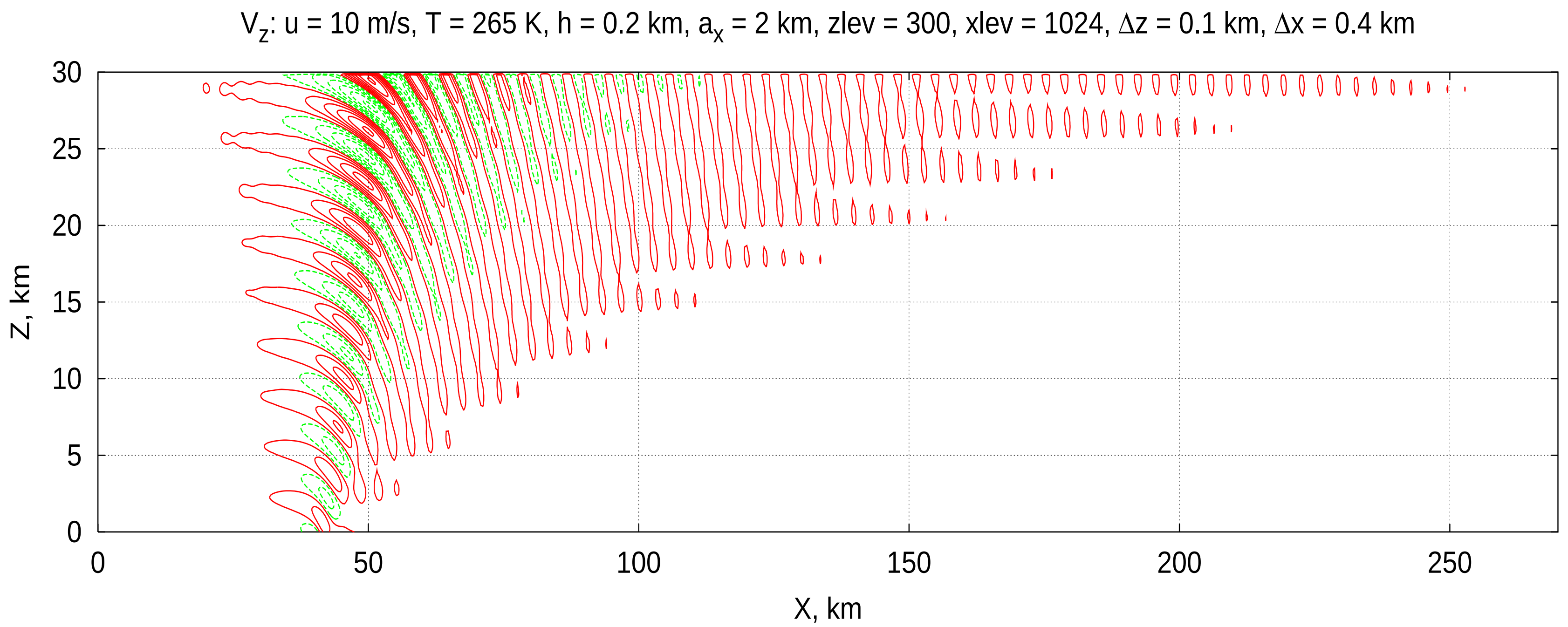
SOLUTION EXAMPLES

I.

Stationary 2D orographic flow over 1D mountain ridge

Vertical velocity waves

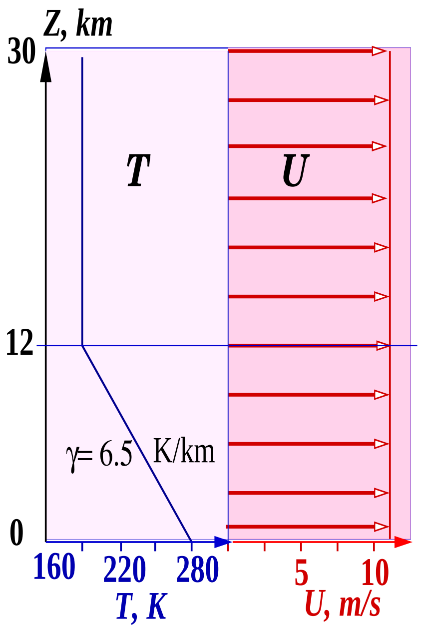
Homogeneous stratification, U = 10 m/s, T = 280 K
Mountain ridge: a_x = 2 km, h = 200 m

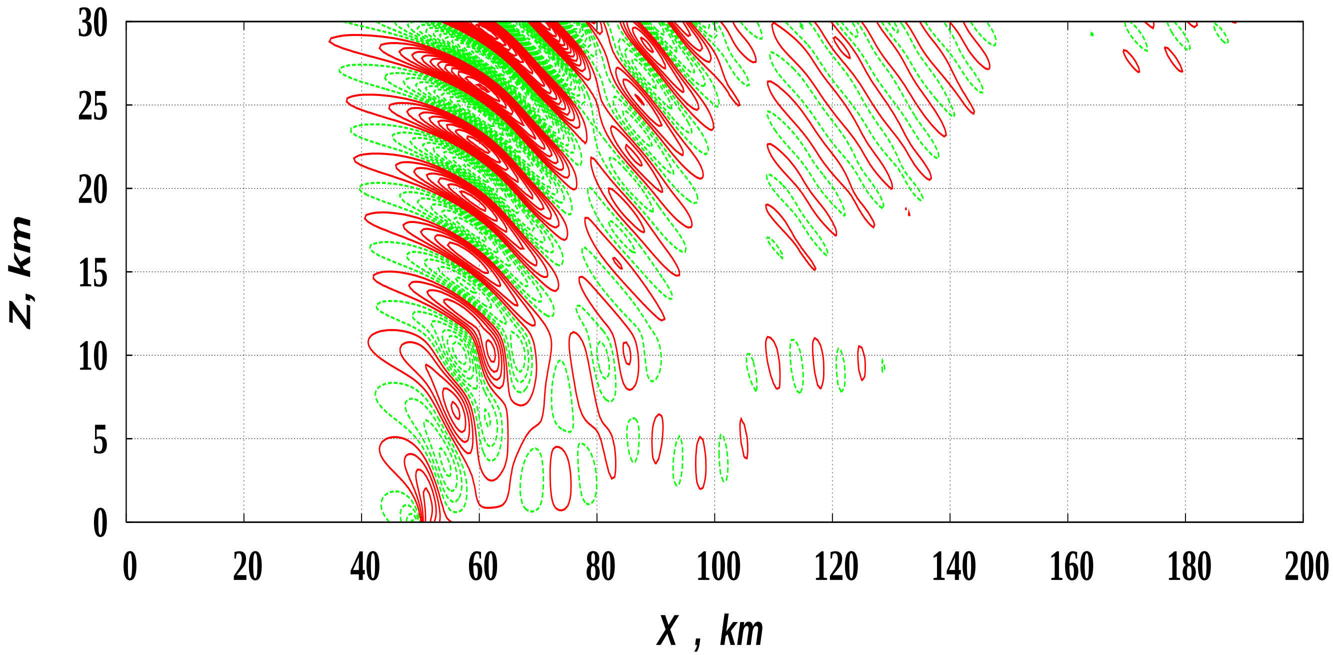


Green - positive, red - negative velocity; $\Delta V_z = 0.1$ m/s

Vertical velocity waves

Refraction and reflection on tropopause.
$U = 12$ m/s, $\gamma = 6.5$ K/km;

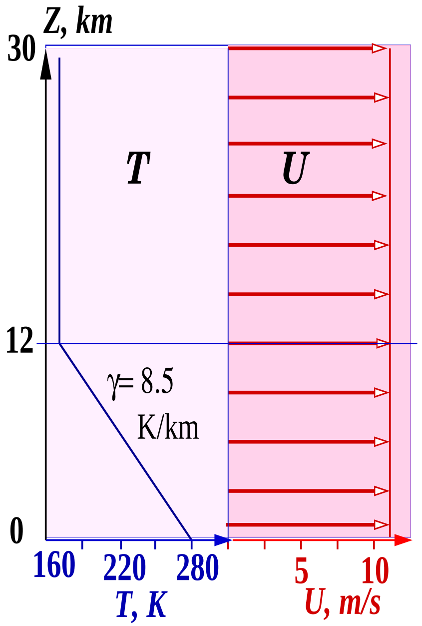


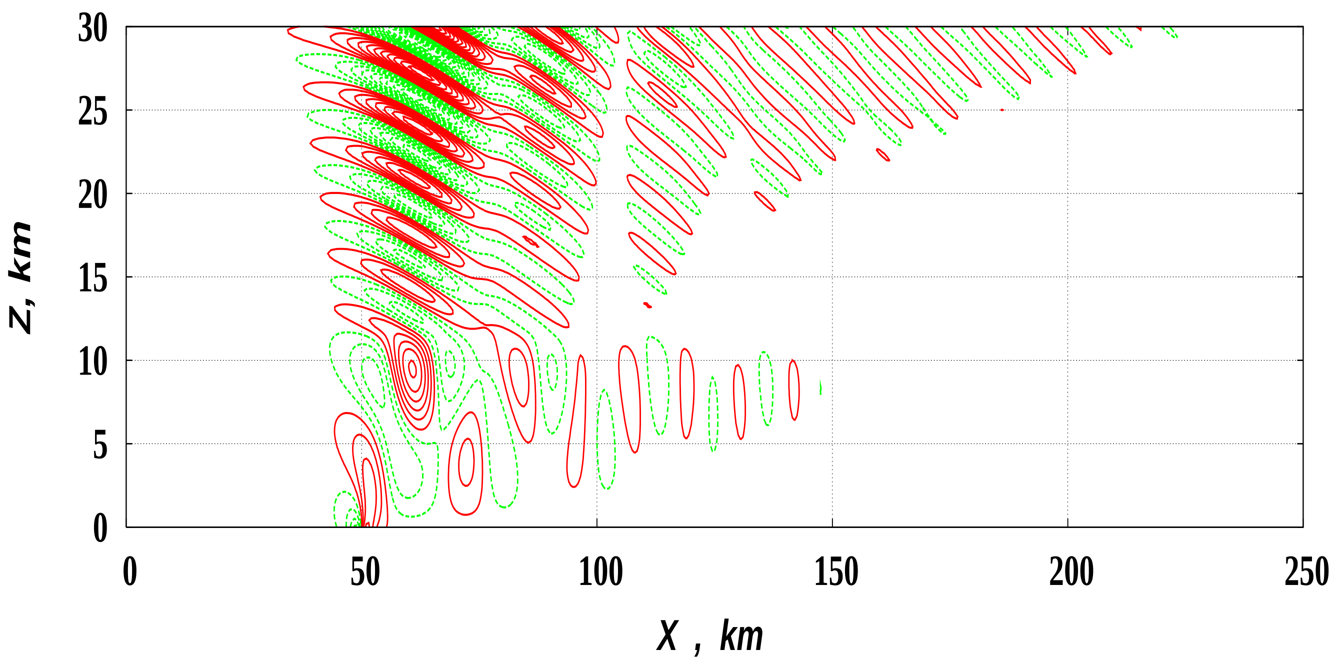


Green - positive, red - negative velocity; $\Delta V_z = 0.1$ m/s

Vertical velocity waves

Refraction and reflection on tropopause.
U = 12 m/s, $\gamma = 8.5$ K/km;

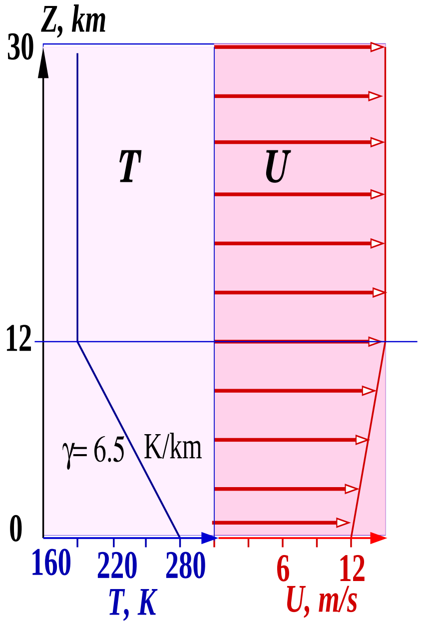


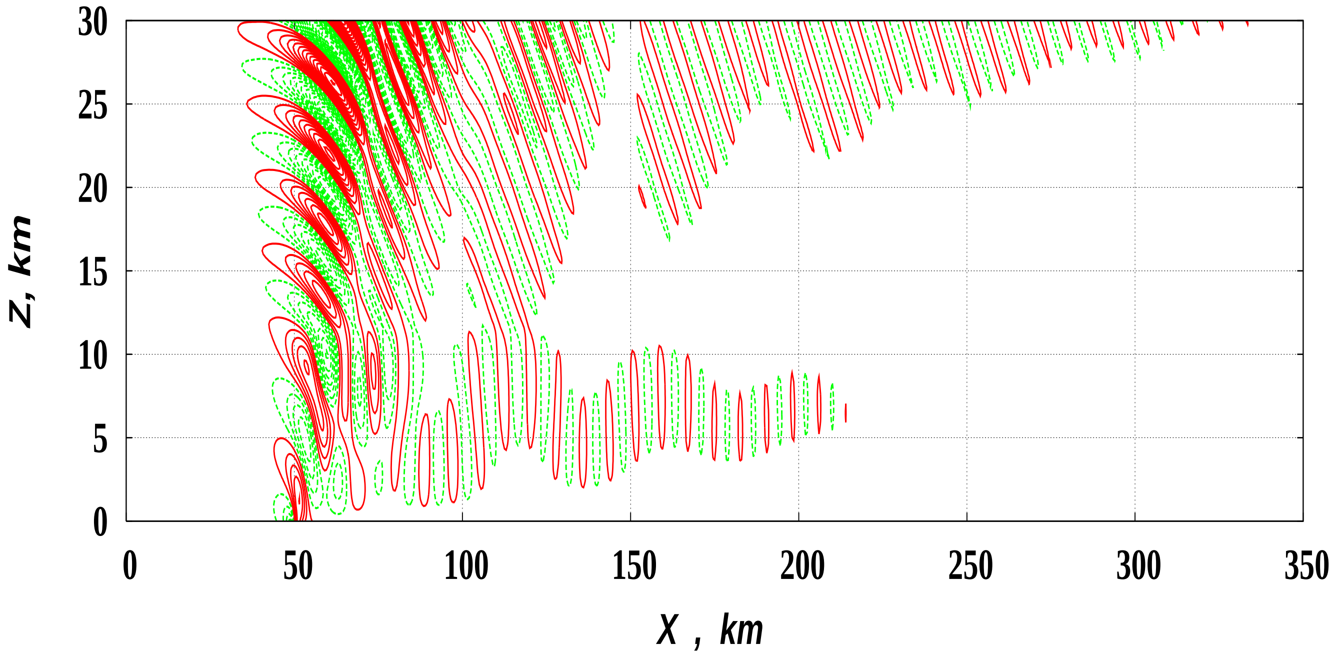


Green - positive, red - negative velocity; $\Delta V_z = 0.1$ m/s

Vertical velocity waves

Refraction and reflection on tropopause in the case of linear wind shear in the troposphere.
U = 12-15 m/s, γ = 6.5 K/km;

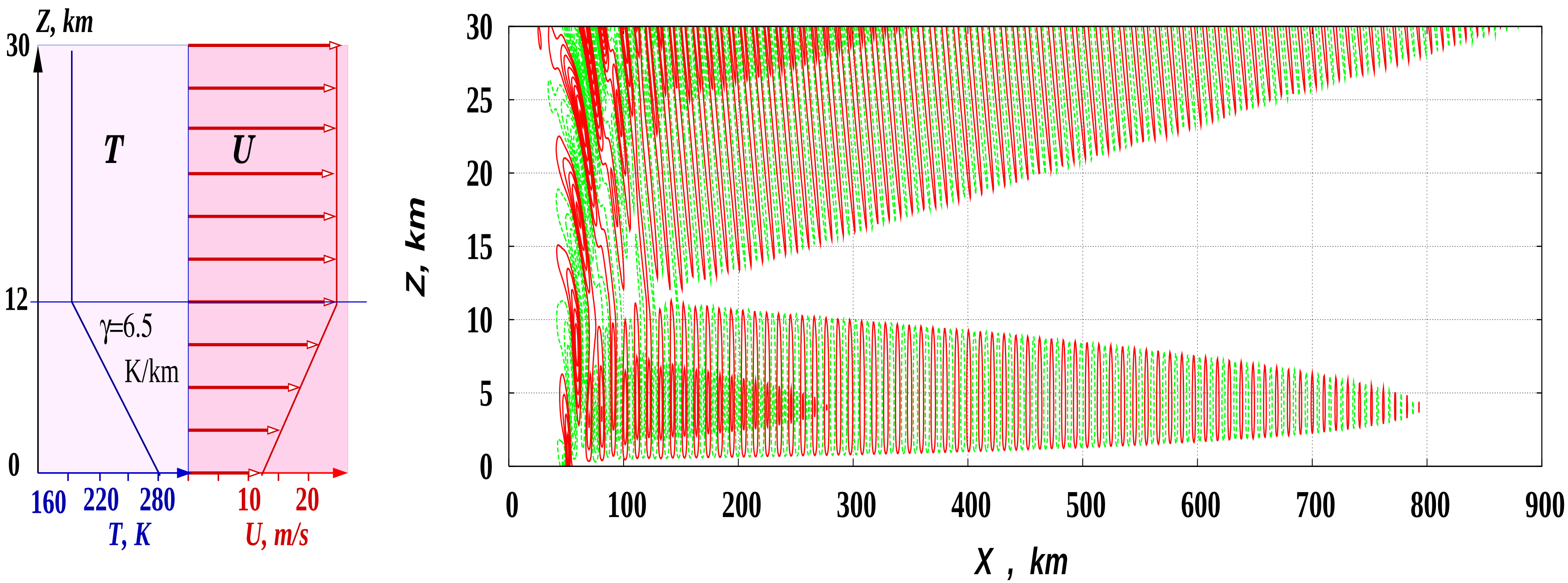




Green - positive, red - negative velocity; $\Delta V_z = 0.1$ m/s

Vertical velocity waves

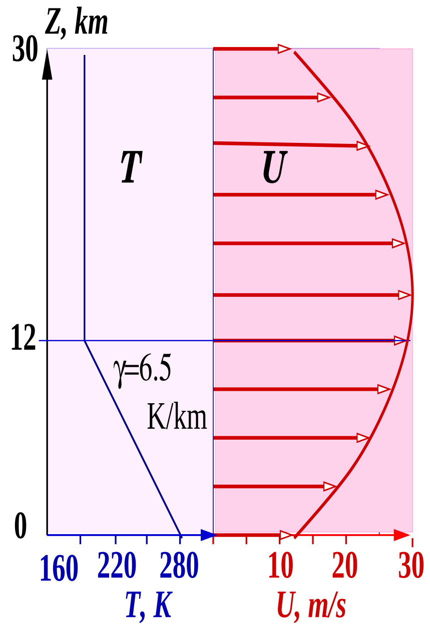
Refraction and reflection on tropopause in the case of linear wind shear in troposphere.
U = 12-24 m/s, $\gamma = 6.5$ K/km;

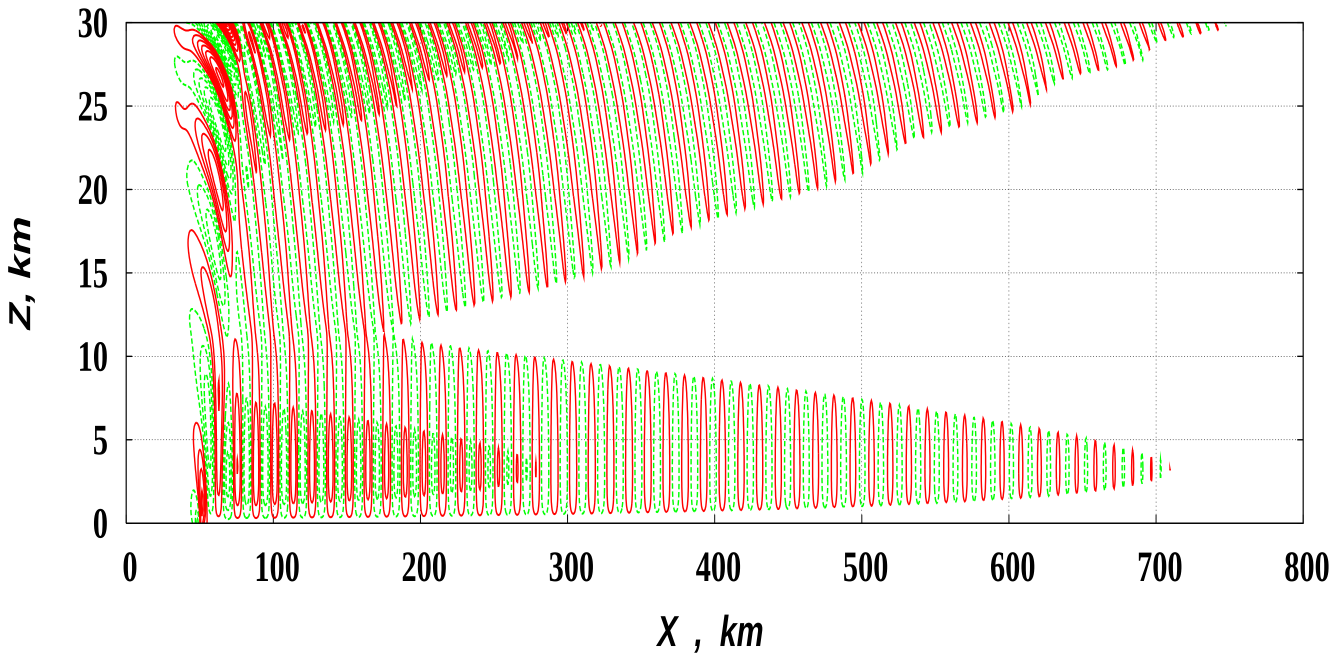


Green - positive, red - negative velocity; $\Delta V_z = 0.1$ m/s

Vertical velocity waves

Hyperbolic wind, U = 10-30 m/s, γ = 6.5 K/km

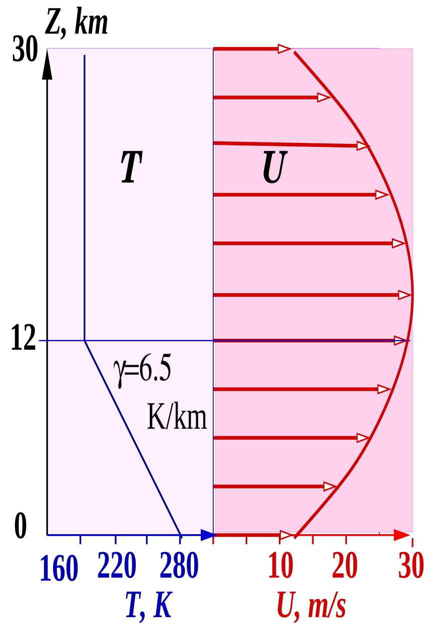


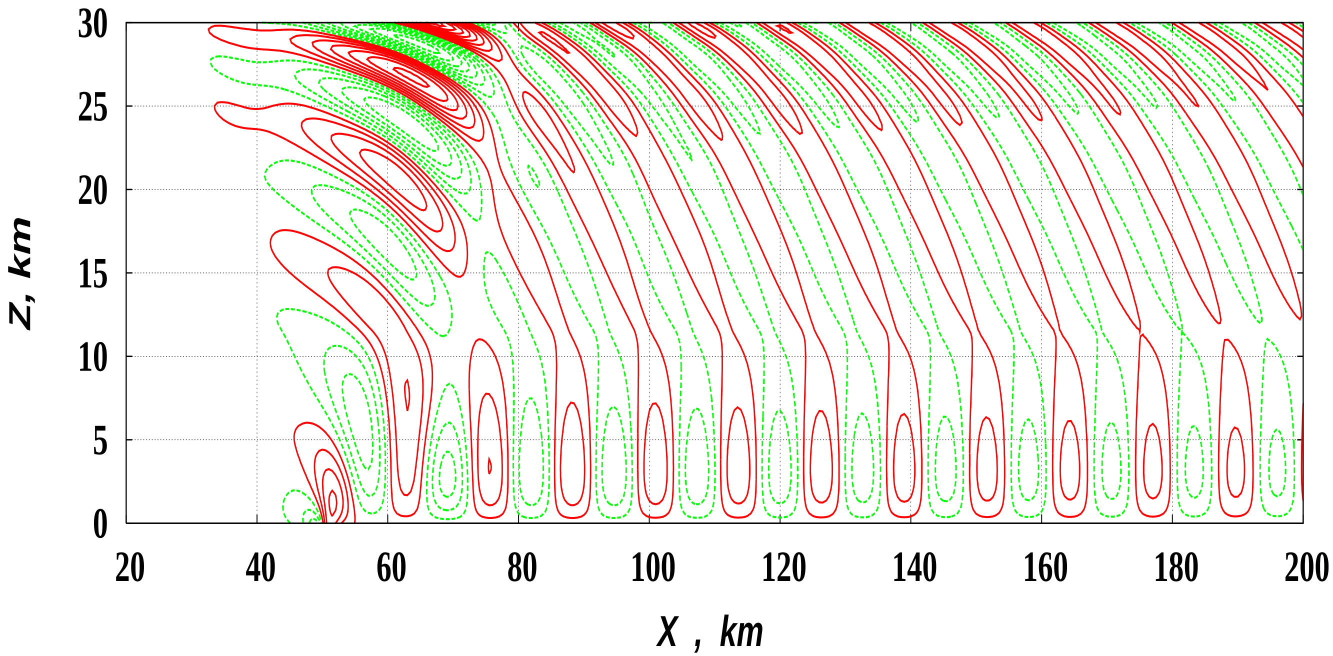


Green - positive, red - negative velocity; $\Delta V_z = 0.1$ m/s

Vertical velocity waves

Hyperbolic wind, U = 10-30 m/s, γ = 6.5 K/km;





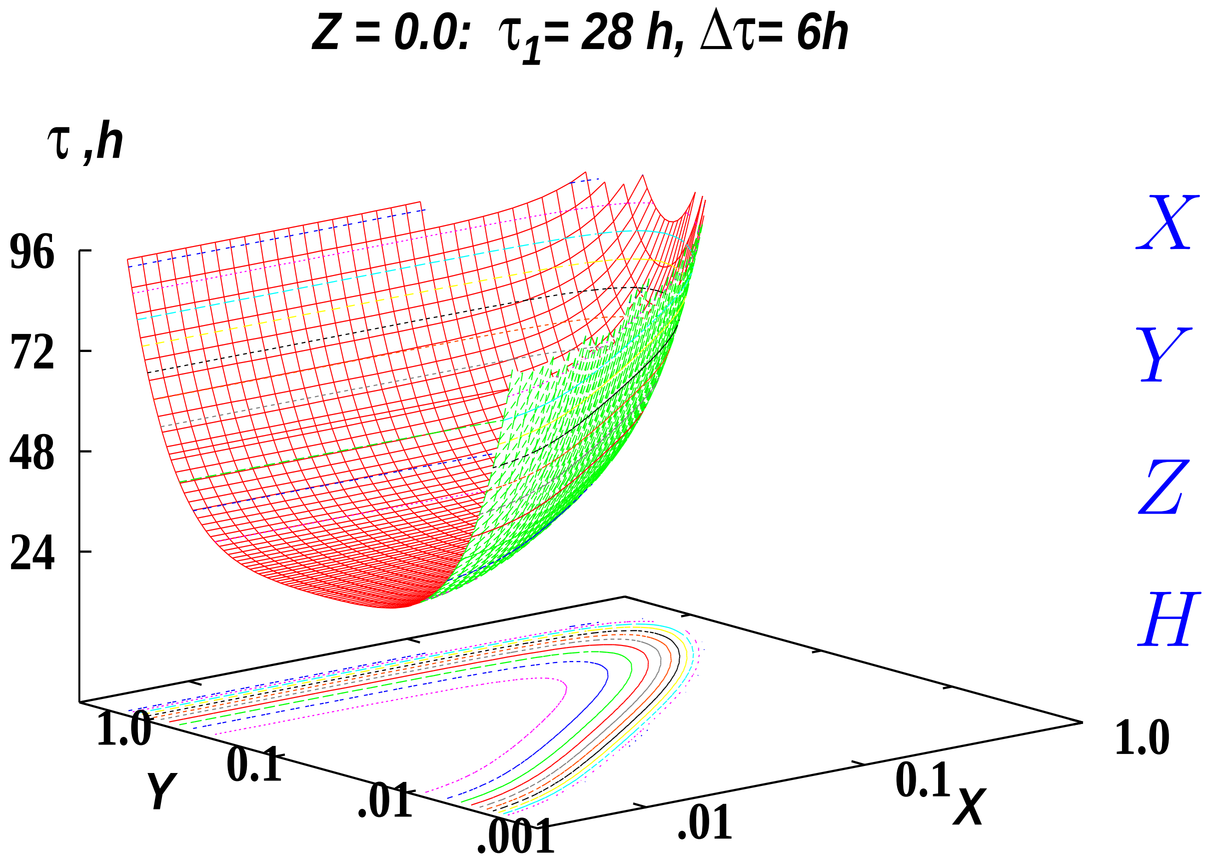
Green - positive, red - negative velocity; $\Delta V_z = 0.1$ m/s

SOLUTION EXAMPLES

II.

Baroclinic instability of long waves

E-folding time of normal modes in case of constant wind shear $dU/dz = 2$ m/s/km



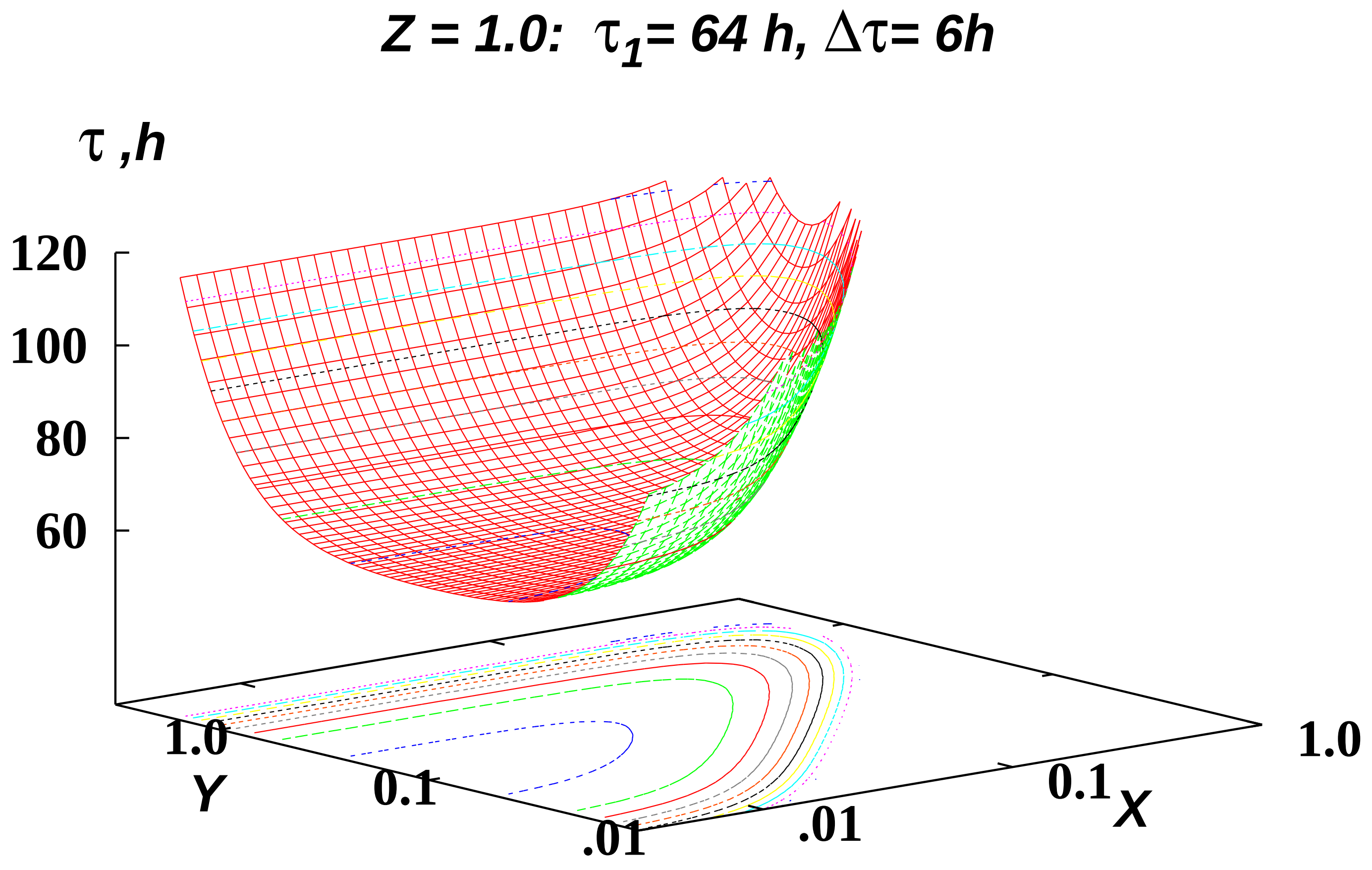
$X = Hk_x$

$Y = Hk_y$

$Z = Hk_z = 0$

$H = 10$ km
- scale height

As previous Fig., except $Z = 1$ ($L_z \sim 10$ km)



3 CONCLUSIONS

Though the numerical scheme was initially developed for test purpose, its actual application area is wider:

Investigation of specific details of orographic flows for complex wind and temperature stratification:

Impact of tropopause, discontinuity of the Brunt-Väisälä frequency, wind shear (including directional shear), boundary layer

Investigation of non-stationary development of linear disturbances, including buoyant instability study

Investigation of the impact of discretization to numerical solution quality:

Vertical discretization (variable Δz)

Accessible time step size and numerical stability issues