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External waves in stream-potential pressure-coordinate dynamics

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Abstract

External wave formation and structure are investigated using external wave equation system, recently developed by authors in frames of 4-dimensional stream-potential dynamics.

The main aim of this presentation is to demonstrate that previous understanding on atmospheric external waves has been rather misleading.

1. Definitions

3D Velocity $\mathbf{v} = \mathbf{e}_\alpha v^\alpha$

$v^x = dx/dt,\ v^y = dy/dt,\ v^p = \omega = dp/dt$

Decomposition to potential and solenoidal parts $\mathbf{v} = \mathbf{v}^0 + \mathbf{v}^s$

Flow potential $\mathbf{v}^0 = \nabla\psi^0$

Stream potential (vector) $\mathbf{v}^s = \nabla \times \boldsymbol{\psi}$

Velocity divergence $\xi^0 = \nabla \cdot \mathbf{v}$

Vorticity $\boldsymbol{\xi} = \nabla \times \mathbf{v}$

Absolute vorticity $\boldsymbol{\Xi} = \mathbf{i}_z f + \boldsymbol{\xi}$

Nonhydrostatic geopotential splitting

$gz(x,y,p,t) =$

Background $= g\bar{z}(p)$

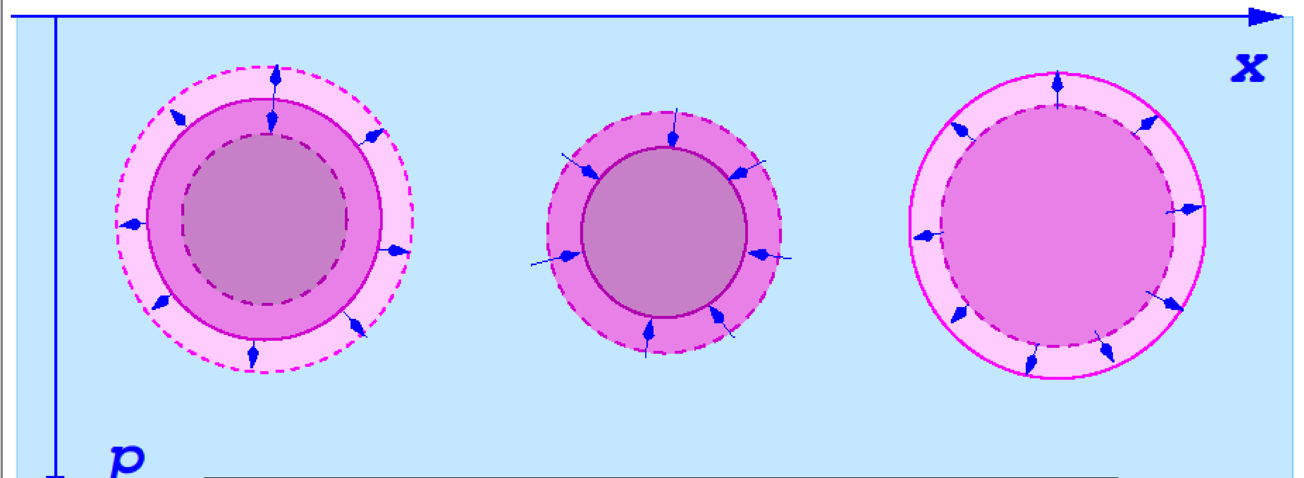
Thermal component $+gz^T$, $\quad gz^T = R\int_p^{p_s(x,y,t)} T'(x,y,p',t)\frac{dp'}{p'}$

Barotropic component, rising from surface pressure fluctuation $+C^2\chi(x,y,t)$

Nonhydrostatic component $+gz'(x,y,p,t)$

Scale height $H = RT/g$

Isochoric sound speed on surface $C^2 = g\bar{H}_s = R\bar{T}_s$



Compressible acoustic oscillation

2. External wave equations

Describe joint evolution of generalized geopotential $\varphi = gz' + C^2\chi + \frac{\mathbf{v}^2}{2}$ and flow potential ψ^0

$$\frac{1}{C^2}\frac{\partial^2}{\partial t^2}\left(p\frac{\partial\varphi}{\partial p}\right) + \mathcal{L}^0\varphi = -S^T - S^V$$

$$\mathcal{L}^0\psi^0 - \frac{\partial}{\partial t}\left(p\frac{\partial\varphi}{\partial p}\right) = -S_\psi \qquad (1)$$

With Laplacian

$$\mathcal{L}^0 = \frac{\partial}{\partial p}\frac{p^2}{H^2}\frac{\partial}{\partial p} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Boundary conditions at the surface are:

$$\psi_s^0 = 0 \quad \left(p\frac{\partial\varphi}{\partial p}\right)_s = \left(p\frac{\partial \mathbf{v}^2/2}{\partial p}\right)_s$$

Usual traditional lower boundary conditions

$$\frac{\partial\varphi_s}{\partial t} = \frac{g^2}{C^2}\left(p\frac{\partial\psi^0}{\partial p}\right)_s - S_s$$

Additional LBC, describing surface pressure evolution and taking care for mass conservation

Sources for potential subsystem

$$S^V = \nabla\cdot(\boldsymbol{\Xi}\times\mathbf{v}+\mathbf{F}) - \frac{p}{2C^2}\frac{\partial}{\partial p}\frac{\partial^2\mathbf{v}^2}{\partial t^2} \qquad S^T = \nabla_h^2 gz^T$$

$$S_\psi = \frac{p}{2C^2}\frac{\partial}{\partial p}\frac{\partial\mathbf{v}^2}{\partial t}$$

are created by solenoidal flow and temperature distribution

$$S_s = \nabla_h\psi_s^0\cdot\nabla_h\chi \qquad \chi = \frac{\varphi_s - \mathbf{v}^2/2}{C^2}$$

3. Zonal flow

To get more close insight into qualities of equations (1), we will consider the case of zonal flow with steady solenoidal component $\mathbf{e}_x U(p)$ on which a small potential velocity $\mathbf{v}^0$ is added, and with mean temperature $T(p)$

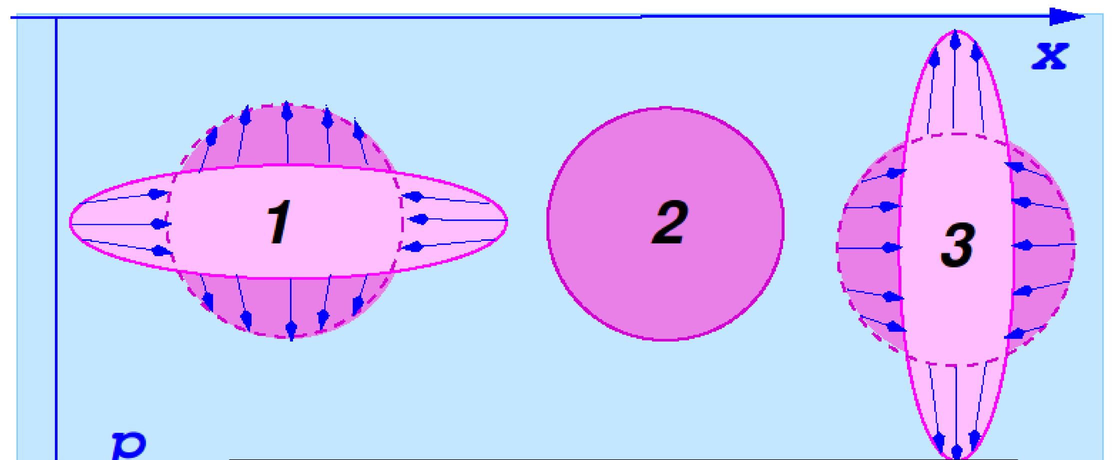
$$\mathbf{V} = \mathbf{e}_x U(p) + \nabla\psi^0(x,y,p,t) \qquad U_q(p) = U_0\left(\frac{p_s}{p}\right)^q$$

$$T = \overline{T}(p) + T'(x,y,p,t)$$

For small perturbation fields sources can be linearized as

$$S^V = S^0(p) + Q(p)\frac{\partial\psi^0}{\partial x} \qquad Q(p) = \frac{\partial f}{\partial y} - \frac{q^2}{H^2}U_q(p)$$

$$S^0(p) = \frac{\partial f}{\partial y}U_q(p) - \frac{q(1-2q)}{H^2}U_q^2(p) \qquad S_\psi = S_s = 0$$



P-space volume preserving hydrostatic oscillation

Depending on the nature of sources, the adjusted and free oscillation regimes can be distinguished

4. Adjusted solution with quasi-stationary sources

If $Q \to 0$ and $T' = T'(x,y,p)$, then $\frac{\partial\varphi}{\partial t} = 0,\ \psi^0 = 0$

And system (1) reduces to single elliptic equation for geopotential

$$\left(\frac{\partial}{\partial p}\frac{p^2}{H^2}\frac{\partial}{\partial p} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi = -\nabla_h^2 gz^T(x,y,p) - S^0(p) \qquad (2)$$

Which has to be solved in stationary case with LBC

$$\varphi_s = 0 \qquad \left(p\frac{\partial\varphi}{\partial p}\right)_s = \left(p\frac{\partial U_q^2/2}{\partial p}\right)_s$$

In particular, for $gz^T = 0$ equation (2) becomes one-dimensional and has explicit solution for H = const.

$$\varphi(p) = \frac{U_q^2(p) - U_q^2(p_s)}{2} \Rightarrow gz' = 0,\ \chi = -\frac{U_q^2(p_s)}{2C^2}$$

For U_0 = 10 m/s this yields surface pressure lowering $\Delta p_s = \chi\, p_s = -0.6$ hPa

and for U_0 = 30 m/s : $\Delta p_s = -5$ hPa

5. Normal mode equations for resonant external waves

If $S0 \to 0$ and $T' = 0$, then system (1) becomes homogeneous with homogeneous LBC

$$\frac{1}{C^2}\frac{\partial^2}{\partial t^2}\left(p\frac{\partial\varphi}{\partial p}\right) + \mathcal{L}^0\varphi = Q(p)\frac{\partial\psi^0}{\partial x}$$

$$\mathcal{L}^0\psi^0 - \frac{\partial}{\partial t}\left(p\frac{\partial\varphi}{\partial p}\right) = 0 \qquad (3)$$

$$\psi_s^0 = 0 \quad \frac{\partial\varphi_s}{\partial t} = \frac{g^2}{C^2}\left(p\frac{\partial\psi^0}{\partial p}\right)_s \quad \left(\frac{\partial\varphi}{\partial p}\right)_s = 0$$

Assuming $Q = \overline{Q(p)} = \overline{\frac{\partial f}{\partial y}} - \frac{q^2}{H^2}\overline{U_q(p)}$ and looking for normal mode solutions in x and t

$$\varphi = \Phi(p)\exp[\mathrm{i}(\nu t - kx)] \qquad \psi^0 = \frac{\Psi(p)}{\nu_0}\exp[\mathrm{i}(\nu t - kx)]$$

equations for normal modes follow

$$\frac{\partial}{\partial p}\left(p^2\frac{\partial\Phi}{\partial p}\right) - \alpha p\frac{\partial\Phi}{\partial p} - (Hk)^2\Phi = \mathrm{i}\beta p\frac{\partial\Psi}{\partial p}$$

$$\frac{\partial}{\partial p}\left(p^2\frac{\partial\Psi}{\partial p}\right) - (Hk)^2\Psi = \mathrm{i}\alpha p\frac{\partial\Phi}{\partial p} \qquad (4)$$

$$\alpha = \nu/\nu_0,\quad \beta = kH^2\overline{Q}/\nu_0,\quad \nu_0 = g/\sqrt{RT}$$

$$\mathrm{i}\alpha\Phi_s = \left(p\frac{\partial\Psi}{\partial p}\right)_s, \quad \left(p\frac{\partial\Phi}{\partial p}\right)_s = 0,\quad \Psi_s = 0$$

6. Solution of normal mode equations (4)

Looking for solution of (4) in form $\Phi = A\left(\frac{p}{p_s}\right)^q,\quad \Psi = B\left(\frac{p}{p_s}\right)^q,$

a homogeneous system for A, B arises

$$\mathrm{i}\alpha qA + [(Hk)^2 - q(q+1)]B = 0$$

$$[q(q+1) - \alpha^2 q - (Hk)^2]A - \mathrm{i}\beta B = 0$$

which yields 4th order characteristic equation for q

$$[q(q+1) - \alpha^2 q - (Hk)^2][q(q+1) - (Hk)^2] - \alpha\beta q = 0$$

If the four roots qj , j = 1,..,4 are found, solution of (4) will be

$$\Phi = \sum_j A_j\left(\frac{p}{p_s}\right)^{q_j} = \mathrm{i}\beta\sum_j\frac{B_j}{q_j(q_j+1) - \alpha^2 q_j - (Hk)^2}\left(\frac{p}{p_s}\right)^{q_j}$$

$$\Psi = \sum_j B_j\left(\frac{p}{p_s}\right)^{q_j} = -\mathrm{i}\alpha\sum_j\frac{q_j A_j}{q_j(q_j+1) - (Hk)^2}\left(\frac{p}{p_s}\right)^{q_j}$$

Obviously, there is no solution for $\alpha = 0$ or $\beta = 0$.

(1) Free waves can't exist in the resting atmosphere (if $\partial f/\partial y \neq 0$ there may exist weak Rossby waves, however)

(2) Stationary adjusted solution is impossible without sourses.

Boundary conditions establish three relations for four amplitudes Aj.

$$\sum\frac{A_j}{(Hk)^2 - q_j(q_j+1)} = 0$$

$$\sum q_j A_j = 0 \qquad \sum\frac{q_j A_j}{(Hk)^2 - q_j(q_j+1)} = 0$$

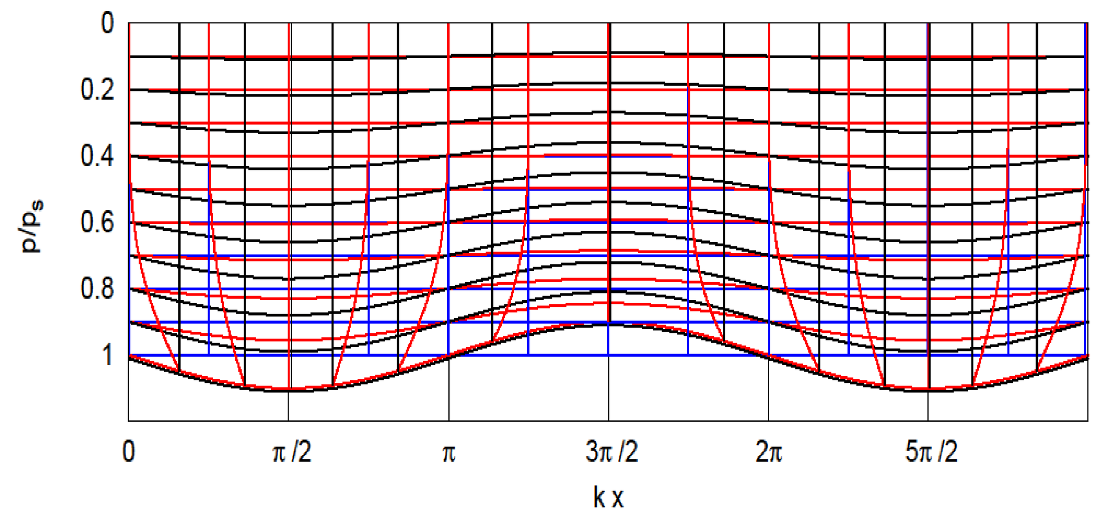
Thus, one amplitude can be chosen free for each given oscillation frequency ν

Finally, looking for solutions of characteristic equation as

$$q = q_k + X,\quad q_k = -1/2 + \sqrt{(Hk)^2 + 1/4}$$

$$X_1^\pm = \frac{\alpha^2 q_k}{2(2q_k+1)}\left(1 \pm \sqrt{1 - \frac{4\beta}{\alpha^3 q_k}}\right) \qquad X_2^\pm = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - b}$$

$$a = \frac{\beta - 2q_k\alpha}{q_k(\alpha^3 q_k - \beta)},\ if\ \alpha, q_k << 1 \qquad b = -\frac{\beta}{\alpha^3 q_k - \beta},\ if\ \alpha, q_k << 1$$



Volume preserving external oscillation