Lecture Notes

NONHYDROSTATIC FORMULATION OF THE HIRLAM

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October 1998

I. INTRODUCTION

1. **Nonhydrostatic effects in shorter meso-scale.**

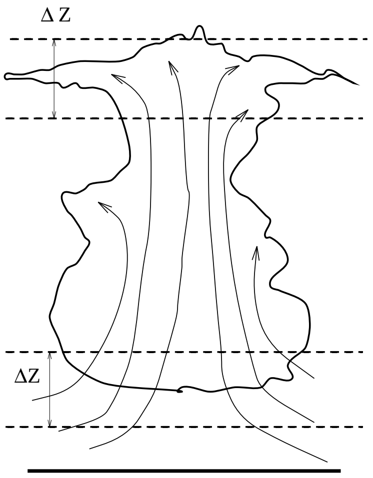
Hydrostatic approximation in equation of vertical development:

$$\rho \frac{\mathrm{d}w}{\mathrm{d}t} = -g\rho - \frac{\partial p}{\partial z} \quad \Rightarrow \quad g\rho + \frac{\partial p}{\partial z} = 0 \ .$$

This approximation is not valid for large vertical accelerations:

– convection:

$$\mathbf{\frac{dw}{dt} \sim 0.1 - 1m/s^2}$$



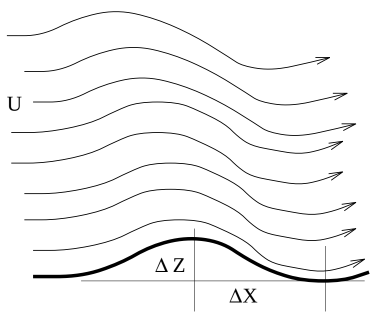
$$\frac{\mathrm{d}w}{\mathrm{d}t} \sim w\frac{\partial w}{\partial z} \sim \frac{(\Delta w)^2}{\Delta z}$$

$\Delta w \sim 10(50) m/s$,

$\Delta z \sim 1km$

– resolved short-scale orographic disturbances:

$$\mathbf{\frac{dw}{dt} \sim 0.01 - 1m/s^2}$$



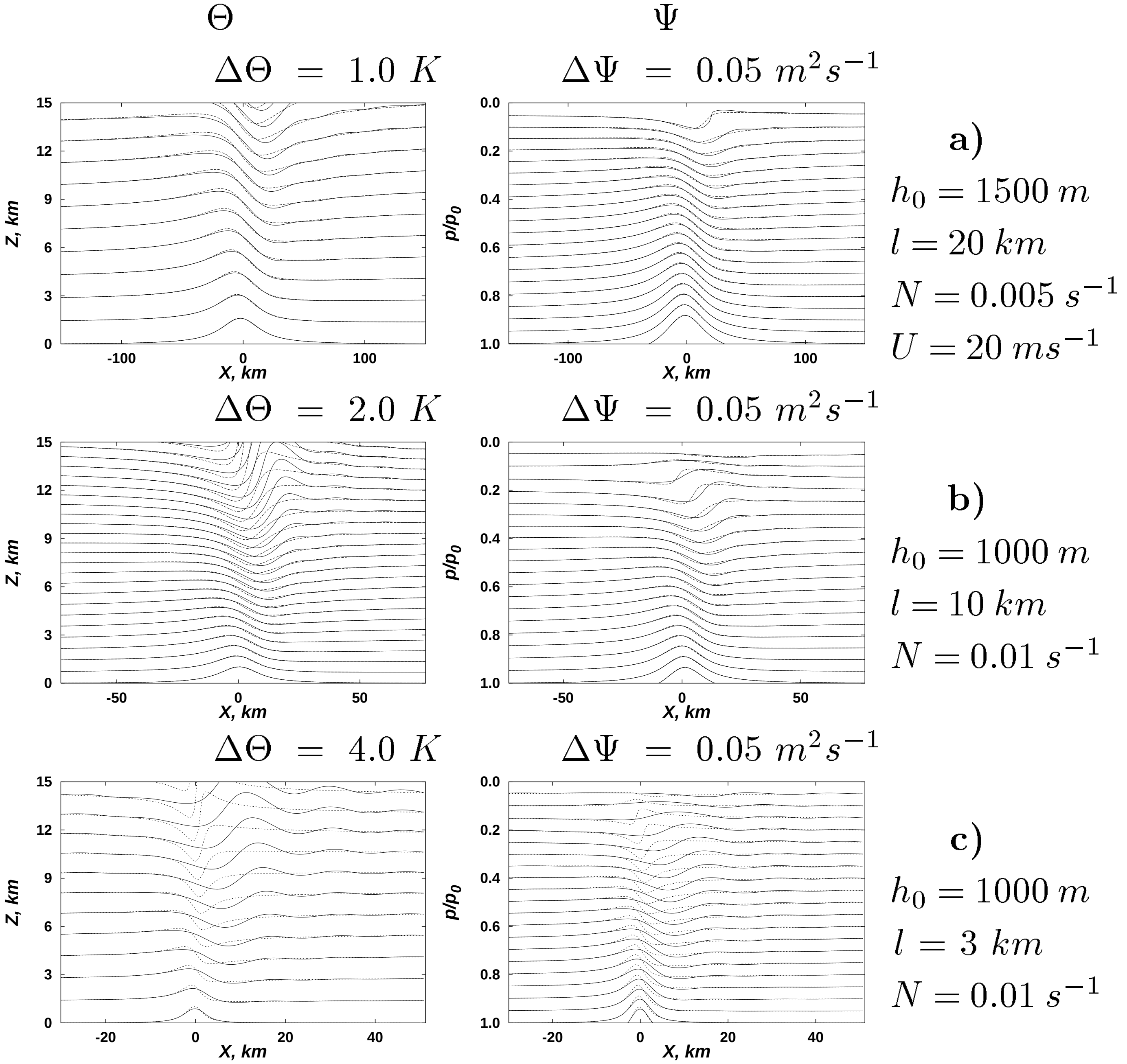
$$\frac{\mathrm{d}w}{\mathrm{d}t} \sim \Delta u \frac{\Delta w}{\Delta z}, \Delta w \sim \frac{\Delta u \Delta z}{\Delta x}$$

$\Delta u \sim 10m/s,\ \Delta x \sim 0.1{-}1km,$

$\Delta z \sim 0.1 - 1km$

Flow modelling over bell-shaped orography

demonstrates that nonhydrostatic and hydrostatic wave–pattern differ essentially for horizontal scales $l < 10 - 20$ km.



Moreover, as it is proved theoretically, the hydrostatic model is not capable of some nonhydrostatic phenomena description (trapped lee waves (Durran 1990) are an example).

2. **Pressure-related coordinates at nonhydrostatic modelling (What is their advantage?)**

Initially pressure coordinates and pressure-related coordinates were developed for **HS** modelling purpose:

Eliassen, 1949: *isobaric* or p–*coordinates*

Phillips, 1957: *sigma-coordinates*, $\sigma = \; p/p_s$

Hybrid- or η–*coordinates* (ECMWF, HIRLAM):

$$\eta = \frac{p}{\mu(p)p_0 + [1 - \mu(p)]p_s} \; , \qquad 0 \; \leq \; \mu(p) \; \leq \; 1$$

Advantages:

– natural coordinates at sounding and analysis

– simplification of equations due to the non-divergent (”anelastic” in the pressure–space terms) nature of the flow:

$$\nabla_p \cdot \mathbf{v} + \frac{\partial \omega}{\partial p} \; = \; 0 \; , \qquad \omega \; \equiv \; \frac{\mathrm{d}p}{\mathrm{d}t}$$

How to be at the entering of the NH -domain? :

A) leave the pressure coordinates in favour of the ordinary Cartesian coordinates?

B) develop nonhydrostatic dynamics in the pressure-coordinate representation?

The second alternative is attractive. Yet for its correctness the pressure must be a **monotonous function of** z. From the vertical development equation a restriction follows:

$$\frac{\partial p}{\partial z} = -\rho\left(g + \frac{\mathrm{d}w}{\mathrm{d}t}\right) < 0 \;\Rightarrow\; \frac{\mathrm{d}w}{\mathrm{d}t} > -g$$

That is, **transition from z to p–coordinates (and application of p–space concept) is correct so far, as vertical accelerations do not exceed the gravitational acceleration!**

There are developed different versions of **NH** p–space dynamics.

Anelastic models: Miller 1974 and Miller and Pearce 1974, White 1989, Salmon and Smith 1994.

Exact (non-simplified) **NH** model: Rõõm 1989, 1990.

Elastic acoustically filtered **NH** model: Rõõm 1997, 1998.

The anelastic model by Salmon and Smith
(which coincides in essence with the White model) **is employed for the development of the NH HIRLAM.**
Details of this model are discussed further.

II. ANELASTIC MODEL IN PRESSURE-RELATED COORDINATES

3. **Anelastic model in p–coordinates. Adiabatic formulation** (Eq. *"NEW"* are the ones which the hydrostatic dynamics lacks)

$$(NEW:)\qquad \frac{\mathrm{d}w}{\mathrm{d}t} = \frac{p}{H}\frac{\partial\phi}{\partial p} + P_w + K_w, \tag{1a}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla_p(\varphi + \phi) - f\mathbf{k}\times\mathbf{v} + \mathbf{P}_\mathbf{v} + \mathbf{K}_\mathbf{v} \tag{1b}$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{gH}{C_p}\frac{\omega}{p} + P_T + K_T\ , \tag{1c}$$

$$\nabla_p\cdot\mathbf{v} + \frac{\partial\omega}{\partial p} = 0\ , \tag{1d}$$

$$\frac{\mathrm{d}p_s}{\mathrm{d}t} = \omega|_{p_s} \tag{1e}$$

$$\frac{\partial\varphi}{\partial p} = -\frac{gH}{p} \tag{1f}$$

$$(NEW:)\qquad \frac{w}{H} = -\frac{\omega}{p} - \frac{1}{R}\frac{\mathrm{d}s}{\mathrm{d}t} \tag{1g}$$

$H = RT/g$	–	height–scale
$w = dz/dt$	–	vertical velocity
$\mathbf{v} = \mathbf{i}\mathrm{u} + \mathbf{j}\mathrm{v}$	–	vector of horizontal velocity
ϕ	–	NH component of the geopotential

4. η-coordinate formulation of the anelastic model. Adiabatic case

Let us make following modification in (1):

i. Eliminate ω in favour of w in the continuity equation;

ii. employ hybrid coordinates;

iii. use expansion $\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_\eta + \eta \frac{\partial}{\partial \eta}$;

iv. express horizontal velocity advection via vorticity and energy gradients like in the hydrostatic HIRLAM.

As the result of these transformations, (1) becomes to

$$(1a) \Rightarrow \qquad \frac{\partial w}{\partial t} = F_w + \hat{S}\phi \qquad (2a)$$

$$(1b) \Rightarrow \qquad \frac{\partial u}{\partial t} = F_u - \hat{G}_\lambda \phi \ , \qquad \frac{\partial v}{\partial t} = F_v - \hat{G}_\theta \phi \ , \qquad (2b)$$

$$(1d) \Rightarrow \qquad \hat{G}_\lambda^+ u + \hat{G}_\theta^+ v - \hat{S}^+ w = 0 \quad \left[= \hat{S}^+ \frac{T\dot{s}}{g} \right] \ . \qquad (2d)$$

Equations for temperature T (1c), ground pressure p_s (1e), and **HS** geopotential φ (1f) do not change and will exactly coincide with those of the "standard", **HS** HIRLAM in hybrid coordinates.

$\mathbf{F_v}$ is the right side of the corresponding equation of the **HS** HIRLAM, and (continuous case)

$$F_w = -\mathbf{v} \cdot \nabla_\eta w - \dot{\eta} \frac{\partial w}{\partial \eta} + P_w + K_w$$

Operators $\hat{\mathbf{G}},\ \hat{\mathbf{G}}^+$ are representations of the horizontal pressure–coordinate gradient and divergence in hybrid coordinates:

$$\hat{G}_\lambda \phi = \begin{cases} \frac{1}{h_\lambda}\left(\frac{\partial\phi}{\partial\lambda}\right)_p, & \text{continuous } p\text{–space} \\ \frac{1}{h_\lambda}\left(\frac{\partial\phi}{\partial\lambda} - \frac{1}{m}\frac{\partial p}{\partial\lambda}\frac{\partial\phi}{\partial\eta}\right)_\eta, & \text{continuous } \eta\text{-space} \\ \frac{1}{\overline{h_\lambda}^\lambda}\left[\delta_\lambda\phi - \frac{1}{\overline{\Delta p}^\lambda}\overline{(\delta_\lambda p)\Delta_\eta\overline{\phi}^\lambda}^\eta\right]_{i+1/2,j,k}, & \text{discrete } \eta\text{-space} \end{cases}$$

$$\hat{G}_\theta \phi = \begin{cases} \frac{1}{h_\theta}\left(\frac{\partial\phi}{\partial\theta}\right)_p, & \text{continuous } p\text{–space} \\ \frac{1}{h_\theta}\left(\frac{\partial\phi}{\partial\theta} - \frac{1}{m}\frac{\partial p}{\partial\theta}\frac{\partial\phi}{\partial\eta}\right), & \text{continuous } \eta\text{-space} \\ \frac{1}{\overline{h_\theta}^\theta}\left[\delta_\theta\phi - \frac{1}{\overline{\Delta p}^\theta}\overline{(\delta_\theta p)\Delta_\eta\overline{\phi}^\theta}^\eta\right]_{i,j+1/2,k}, & \text{discrete } \eta\text{-space} \end{cases}$$

Notation here is the standard HIRLAM notation, except

$$m = \frac{\partial p}{\partial\eta}$$

$$\hat{G}^+_\lambda u = \begin{cases} \frac{1}{h_\lambda}\left(\frac{\partial u}{\partial \lambda}\right)_p, & \text{continuous } p\text{–space} \\ \frac{1}{h_\lambda}\left(\frac{\partial u}{\partial \lambda}\right)_\eta - \frac{1}{m}\frac{1}{h_\lambda}\left(\frac{\partial p}{\partial \lambda}\right)_\eta \frac{\partial u}{\partial \eta}, & \text{continuous } \eta\text{-space} \\ \left[\frac{1}{h_\lambda}\left(\delta_\lambda u - \frac{1}{\Delta p}\overline{\delta_\lambda p \Delta_\eta u}^{\lambda\eta}\right)\right]_{i,j,k}, & \text{discrete } \eta\text{-space} \end{cases}$$

$$\hat{G}^+_\theta v = \begin{cases} \frac{1}{h_\theta}\left(\frac{\partial v}{\partial \theta}\right)_p, & \text{continuous } p\text{–space} \\ \frac{1}{h_\theta}\left(\frac{\partial v}{\partial \theta}\right)_\eta - \frac{1}{m}\frac{1}{h_\theta}\left(\frac{\partial p}{\partial \theta}\right)_\eta \frac{\partial v}{\partial \eta}, & \text{continuous } \eta\text{-space} \\ \left[\frac{1}{h_\theta}\left(\delta_\theta v - \frac{1}{\Delta p}\overline{\delta_\theta p \Delta_\eta v}^{\theta\eta}\right)\right]_{i,j,k}, & \text{discrete } \eta\text{-space} \end{cases}$$

$$\hat{S}\phi \quad = \quad \begin{cases} \frac{p}{H}\frac{\partial \phi}{\partial p}, & \text{continuous } p\text{–space} \\ \frac{p}{mH}\frac{\partial \phi}{\partial \eta}, & \text{continuous } \eta\text{-space} \\ \left(\frac{\overline{g\rho}^\eta}{\overline{m}^\eta}\Delta_\eta \phi\right)_{i,j,k+1/2}, & \text{discrete } \eta\text{-space} \end{cases}$$

$$\hat{S}^+\phi \quad = \quad \begin{cases} \frac{\partial}{\partial p}\frac{p\phi}{H}, & \text{continuous } p\text{–space} \\ \frac{1}{m}\frac{\partial}{\partial \eta}\frac{p\phi}{H}, & \text{continuous } \eta\text{-space} \\ \left[\frac{1}{\Delta p}\Delta_\eta\left(\overline{g\rho}^\eta w\right)\right]_{i,j,k}, & \text{discrete } \eta\text{-space} \end{cases}$$

III. NONHYDROSTATIC EULERIAN SCHEME

5. Poisson equation for NH geopotential height fluctuation

Action on eq. (2d) with $\partial/\partial t$ yields the Poisson equation for ϕ (Discrete η-coordinate case, all relations are defined at internal grid-points $\{i, j, k\}$):

$$\mathcal{L}\phi = A \ , \tag{3}$$

$$\mathcal{L} = \hat{G}_\lambda^+ \hat{G}_\lambda + \hat{G}_\theta^+ \hat{G}_\theta + \hat{S}^+ \hat{S} \ ,$$

$$A = \hat{G}_\lambda^+ F_u + \hat{G}_\theta^+ F_v - \hat{S}^+ F_w + \frac{\partial \hat{G}_\lambda^+}{\partial t} u + \frac{\partial \hat{G}_\theta^+}{\partial t} v + \frac{\partial \hat{S}^+}{\partial t} \frac{H\omega}{p}$$

$$F_w = -\left(\overline{\frac{\overline{u}^\eta}{\overline{h_\lambda}^\lambda} \delta_\lambda w}^\lambda + \overline{\frac{\overline{v}^\eta}{\overline{h_\theta}^\theta} \delta_\theta w}^\theta + m\dot{\eta} \overline{\left(\frac{\Delta w}{\Delta p} \right)}^\eta \right) + P_w + K_w$$

Source function A represents the 3D divergence of **HS** tendency + small contribution (last three terms), proportional to tendencies of operators.

In the continuous p-coordinate representation $\mathcal{L}$ would be:

$$\mathcal{L}\phi = \nabla_p^2 \phi \ + \ \frac{\partial}{\partial p} \left(\frac{p^2}{H^2} \frac{\partial \phi}{\partial p} \right)$$

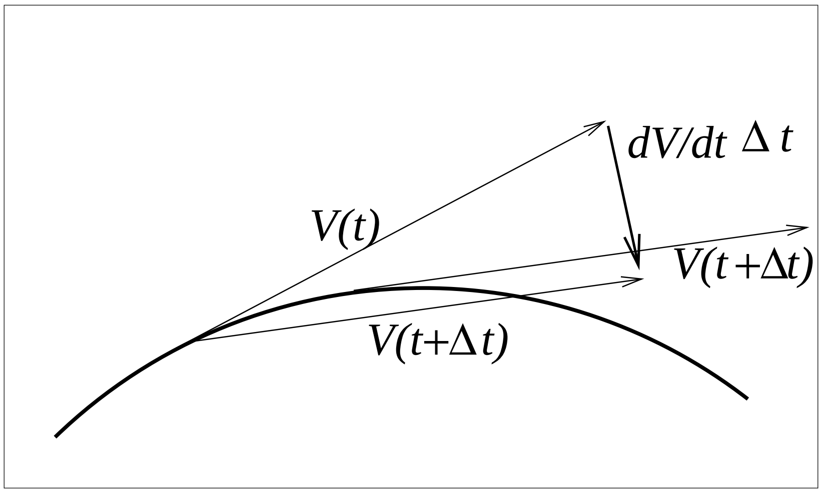
6. Boundary conditions for ϕ

i. Lateral boundaries: Zero normal gradient (no acceleration due to **NH** geopotential)

$$\left(\frac{\partial\phi}{\partial n}\right)_L = 0 \quad (= \Gamma_L(\phi) \quad \text{as option }) \tag{4a}$$

ii. Boundary condition on the bottom:

Normal gradient $\partial\phi/\partial\eta$ at the bottom ($\eta = 1$) must maintain air particles at free–slip on the surface



The required condition can be obtained, acting with $\frac{\partial}{\partial t}$ on the free–slip condition $w|_{\eta=1} = \mathbf{v}|_{\eta=1} \cdot \nabla Z$. The result reads (after some 'algebra'; the continuous η-space representation is assumed)

$$\left(\frac{\partial\phi}{\partial\eta}\right)_{\eta=1} = \Gamma_s^0 + \Gamma_s^1(\phi) \ , \tag{4b}$$

$$\Gamma_s^0 = \left\{\frac{mH\mathcal{F}}{p[1+(H\nabla p/p)^2]}\right\}_{\eta=1}, \Gamma_s^1(\phi) = \left\{\frac{mH^2\nabla p\cdot\nabla\phi}{p^2[1+(H\nabla p/p)^2]}\right\}_{\eta=1}$$

$$\mathcal{F} = -F_w - H\nabla p\cdot\mathbf{F_v}/p \ .$$

If horizontal scale is $>$ 5000 m – 1 km, then (4b) can be approximated as homogeneous ($\Gamma^0, \ \Gamma^1 \ \to \ 0$).

iii. Boundary condition at the top follows from the requirement that ϕ is finite at $\eta \to 0$, if the source function A is finite. This condition can be formulated quantitatively in the Fourier representation by the horizontal coordinates.
We define the main part of $\mathcal{L}$ as its horizontally averaged component:

$$\overline{\mathcal{L}} = \left(\frac{1}{\langle h_\lambda \rangle}\frac{\partial}{\partial \lambda}\right)^2 + \left(\frac{1}{\langle h_\theta \rangle}\frac{\partial}{\partial \theta}\right)^2 + \frac{1}{\langle m \rangle}\frac{\partial}{\partial \eta}\frac{\langle p/H \rangle^2}{\langle m \rangle}\frac{\partial}{\partial \eta}$$

where $\langle \cdot \rangle$ is the operation of horizontal averaging:

$$\{k\} : \qquad \langle a \rangle = \frac{1}{NlatNlon}\sum_{i,j} a_{ijk}$$

Let us consider the main-part approximation to the exact Poisson equation for ϕ

$$\overline{\mathcal{L}}\phi = A$$

Its Fourier transform in x, y reads

$$\frac{1}{\langle m \rangle}\frac{\partial}{\partial \eta}\frac{\langle p/H \rangle^2}{\langle m \rangle}\frac{\partial \tilde{\phi}_\kappa}{\partial \eta} - \kappa^2 \tilde{\phi}_\kappa = \tilde{A}_\kappa$$

where $\tilde{\phi}_\kappa$, $\tilde{A}_\kappa$ are Fourier amplitudes of ϕ and A, and $\kappa = \{\kappa_x, \kappa_y\}$ is the wave-vector, whereas $\kappa^2 = (\kappa_x)^2 + (\kappa_y)^2$ is the Fourier transform of the horizontally averaged Laplacian.

The upper boundary condition for ϕ_κ is

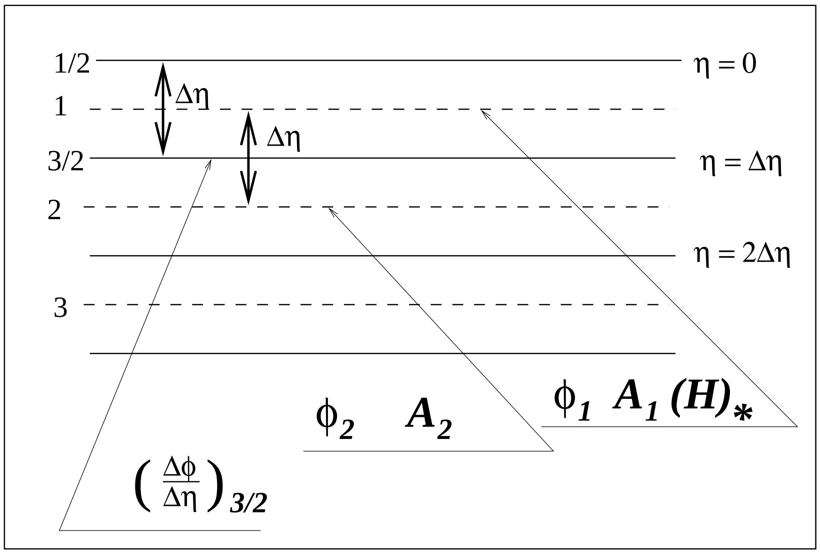
$$\eta_* \left(\frac{\partial \tilde{\phi}_\kappa}{\partial \eta} \right)_{\eta_*} - \mu_\kappa \tilde{\phi}_\kappa(\eta_*) = \frac{\langle H \rangle_*^2 \tilde{A}_\kappa(\eta_*)}{\mu_\kappa + 1} \ , \qquad (4c)$$

where

$$\mu_\kappa = \sqrt{1/4 + \langle H \rangle_*^2 \kappa^2} - 1/2 \ ,$$

η_* is a level near the top and $\langle H \rangle_*$ is mean value of $\langle H \rangle$ above that level.

In the discrete model, η_* corresponds to the level $k = 3/2$, $\eta_* = \Delta\eta$.



The discrete approximation (wave index κ is omitted) to (4c) is

$$\hat{L}\tilde{\phi} \equiv (\tilde{\phi}_{ij2} - \tilde{\phi}_{ij1}) - \frac{\mu_{ij}}{2}(\tilde{\phi}_{ij1} + \tilde{\phi}_{ij2}) = \langle H \rangle_1^2 \frac{\tilde{A}_{ij1} + \tilde{A}_{ij2}}{2(\mu_{ij} + 1)} \equiv \hat{G}\tilde{A} \ .$$

7. Solution of elliptic equation for ϕ

Iterative algorithm is employed, which supports the fast Fourier cosine-transform (**FFCT**).

Initial equation and boundary conditions are presented as

$$\overline{\mathcal{L}}\phi = A - \mathcal{L}'\phi \ ,$$

$$\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=1} = \Gamma_s^0 + \Gamma_s^1 \phi \, , \ \left(\frac{\partial \phi}{\partial n}\right)_L = \Gamma_L^0 + \Gamma_L^1 \phi \, , \ \hat{L}\phi = \hat{G}(A - \mathcal{L}'\phi) \, .$$

Here $\mathcal{L}'$ is the perturbation component of the elliptic operator,

$$\mathcal{L}' = \mathcal{L} - \overline{\mathcal{L}}$$

which becomes zero for flat, plain ground and horizontally homogeneous stratification: $Z = 0$, $H = H(\eta)$, $p_s = p_0$.

The idea is to use the iterative process, in which the ith approximation ϕ^i is solution of the equation

$$\overline{\mathcal{L}}\phi^i = A - \mathcal{L}'\phi^{i-1} \ ,$$

$$\left(\frac{\partial \phi^i}{\partial \eta}\right)_{\eta=1} = \Gamma_s^0 + \Gamma_s^1 \phi^{i-1} \ , \ \left(\frac{\partial \phi^i}{\partial n}\right)_L = \Gamma_L^0 + \Gamma_L^1 \phi^{i-1} \ ,$$

$$\hat{L}\phi^i = \hat{G}(A - \mathcal{L}'\phi^{i-1}) \ ,$$

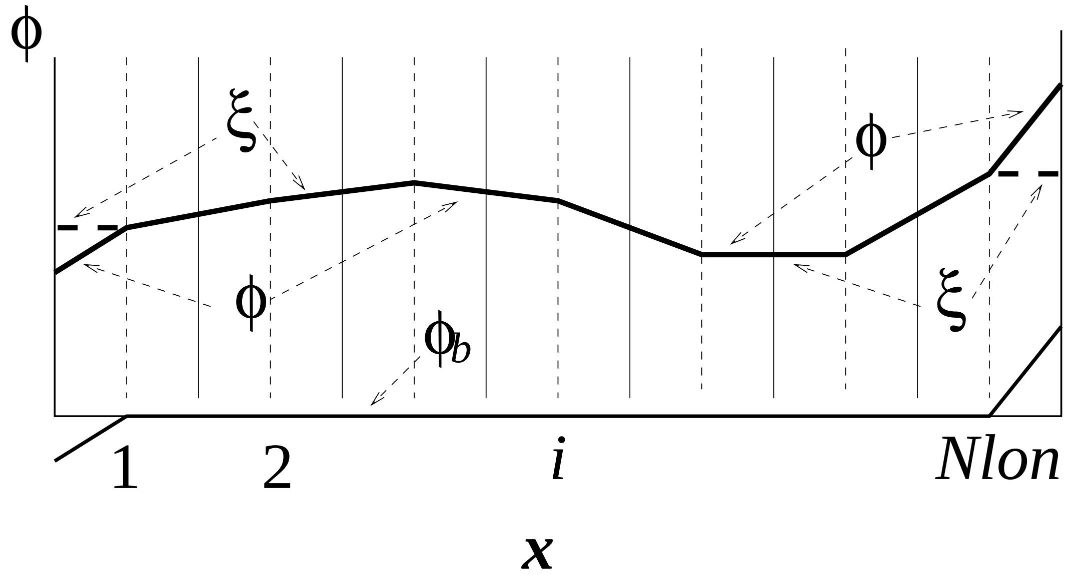
starting with

$$\phi^0 = 0 \ .$$

Application of the FFCT requires expansion (Winninghoff 1968, Williams 1969)

$$\phi^i = \xi^i + \phi_b^i$$

where ξ^i has zero normal gradient at lateral boundary and ϕ_b^i is zero in the internal points of the domain. Thus,



$$\left(\frac{\partial \phi_b^i}{\partial \eta}\right)_1 = \Gamma_s^0 + \Gamma_s^1 \phi^{i-1}, \left(\frac{\partial \phi_b^i}{\partial n}\right)_L = \Gamma_L^0 + \Gamma_L^1 \phi^{i-1}, \hat{L}\phi_b^i = \hat{G}(A - \mathcal{L}'\phi^{i-1})$$

$$\overline{\mathcal{L}}\xi^i = A - \overline{\mathcal{L}}\phi_b^i - \mathcal{L}'\phi^{i-1} \equiv A^i ,$$

$$\left(\frac{\partial \xi^i}{\partial \eta}\right)_{\eta=1} = 0, \quad \left(\frac{\partial \xi^i}{\partial n}\right)_L = 0 , \quad \hat{L}\xi^i = 0 ,$$

$$\phi^i = \xi^i + \phi_b^i , \quad \xi^0 = 0 , \quad \phi_b^0 = 0 .$$

As a rule, 4 - 5 iterations are required to compute nonhydrostatic geopotential height fluctuation ϕ/g with error ≤ 1 cm.

8. Numerical scheme

Described algorithm is realized on the variant of HIRLAM 2.5:

$50 \times 50 \times 24$ points; horizontal resolution 11.1 km; area 555×555 km^2.

Modifications concern subroutine 'EULER':
– routine 'DYN', which is called from 'EULER' is modified to include computation of w and $\partial w/\partial t$.
– thereafter routine 'ellipt' is called, which computes ϕ.
– NH tendencies of u, v are added to the hydrostatic counterparts to get full tendencies.

Present NH version assumes the Eulerian time integration (Explicit Leapfrog Scheme). Common Explicit Leapfrog requires a small time step, $\Delta t < \Delta x/C_e$, where $C_e \approx 280$ m/s is the external buoyancy wave (Lamb wave) speed. To increase Δt, external waves are eliminated using the rigid bottom approximation in the $pressure - space$: $\frac{\partial p_s}{\partial t} \rightarrow 0$. Thus, the modelling domain $0 < p \leq \overline{p}_s = p_s|_{t=0}$ is the same for all integration period. The actual surface pressure is considered as an adjusted field and is computed via the boundary value of the NH geopotential fluctuation ϕ at the lowest model surface $\overline{p}_s$:

$$p_s - \overline{p}_s = \left(1 + \frac{\phi}{RT}\right)_{\eta=1} .$$

Such approximation for lower boundary is good so far, as the modelling domain is small (up to 1000 – 2000 km in horizontal).

Yet the problem has temporary nature, as the time–step restriction (and along with it a need to fix the ground pressure) are in practice eliminated in the implicit scheme, which would be a next modification to the present **NH** version anyway.

Lateral boundary relaxation scheme.

The existing boundary relaxation scheme of the **HS** HIRLAM does not work properly in the **NH** modification. It is too inflexible and causes buoyancy wave reflection on lateral boundaries and consequent standing– wave formation near the boundary relaxation zone:

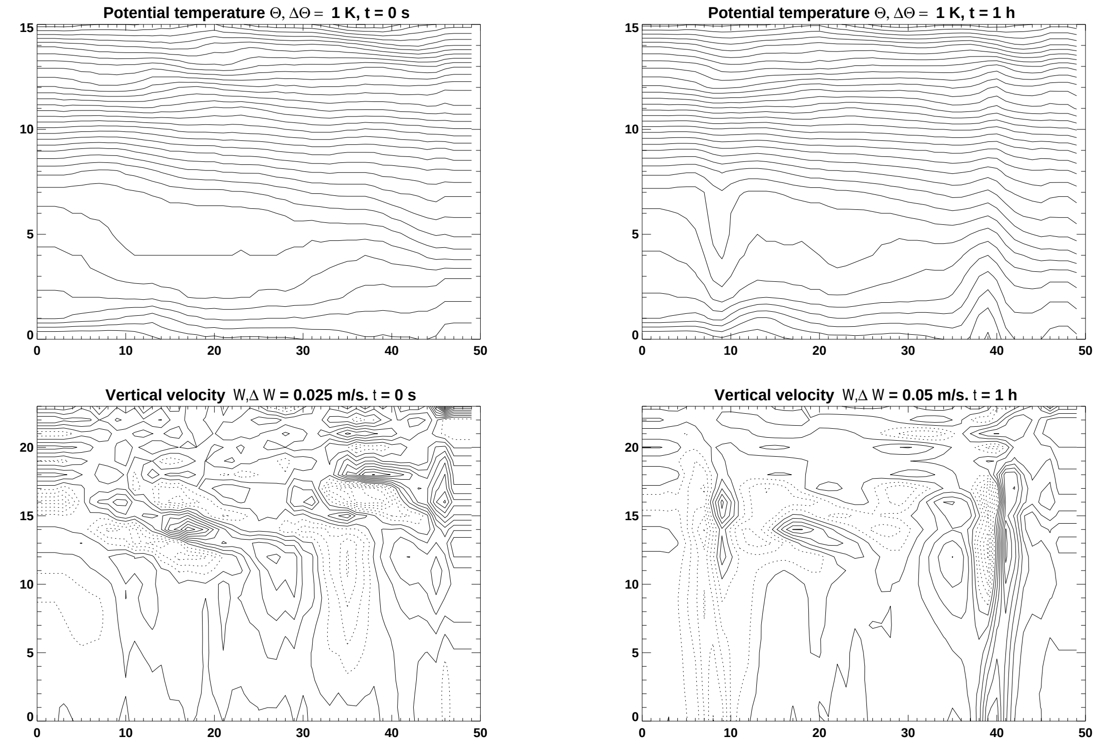


Fig. Potential temperature θ (upper panels) and vertical velocity w (lower panels) in vertical plane $j = 25$ in the initial moment (left panels) and after 1 h (120 steps of integration).

Next figure illustrates what changes undertakes the source function A in the ϕ–equation (3) during a short integration (1 hour of real time), if the common relaxation scheme of the **HS** HIRLAM is used.

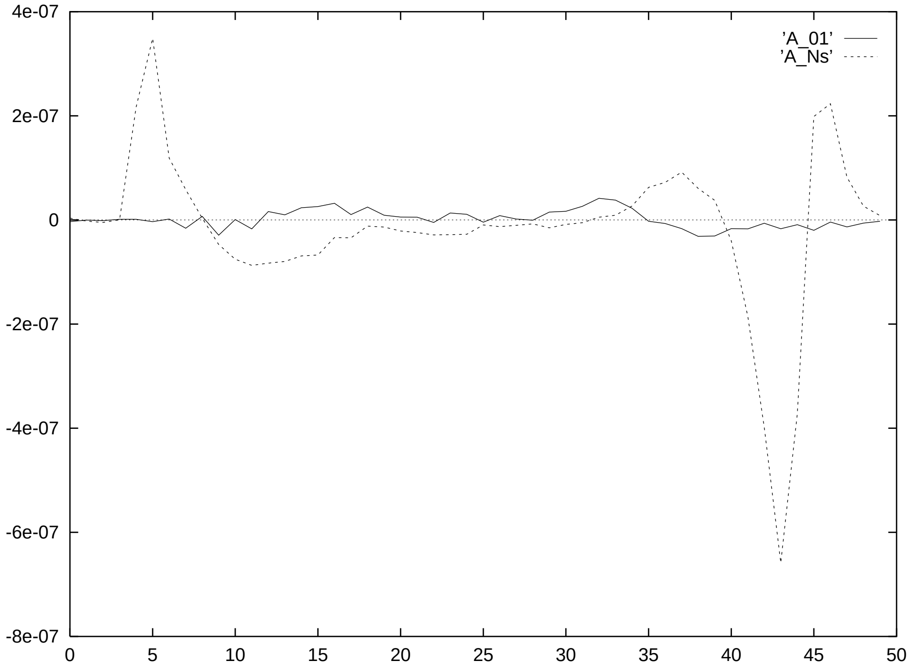


Fig. Source function A along line $k = 13$, $j = 25$ for $istep = 1$ and $istep = 120$. $\Delta t = 30$ s.

There exist two principal schemes to avoid buoyancy wave reflection at lateral boundaries. The first applies **radiative boundary condition** and in this way makes the boundaries transparent to gravity waves (Orlanski 1976, Raymond and Kuo 1984). This approach is, for instance, applied in the **NH** sigma-coordinate models NH3D (Miranda 1990, Miranda and James 1992) and NHAD (Rõõm 1997), and in the **NH** mesoscale model of the MRI (Ikawa and Saito 1991). The other makes use of the Davies-type absorbing layer (Davies 1976), introducing the Newton-Rayleigh friction near boundaries. This relaxation scheme is implemented in the *Lokal-Modell* of the DWD (Doms and Schaettler 1997). The mechanism is similar to the one, used at upper boundary ($\eta \rightarrow 0$) by Klemp and Lilly (1978).

Instead of the rigid relaxation scheme of the **HS** HIRLAM, which applies relaxation scheme in the boundary zone $L - \Delta L < x < L$

$$a^{in}(x) \rightarrow a^{rel}(x) = a^{in}(x) \cdot (1 - w(x)) + a^{ref}(x)w(x)$$

(a is arbitrary field to be relaxed, and $w(x)$ is the given weight-function), the Davies mechanism makes rather use of the Newton–Rayleigh friction

$$\frac{\partial a}{\partial t} = F_a - K(x)(a - a^{ref})$$

where $K(x)$ is a positive friction coefficient:

$$K(x) = \begin{cases} 0 \quad , & x < L - \Delta L \\ c(x - L + \Delta L)^2 \ , & L - \Delta L < x < L \end{cases}$$

and F_a represents the ordinary tendency for a (without friction).

In the case of the **NH** HIRLAM the Davies scheme is more preferable as the **HS** HIRLAM makes already use of the relaxation zone, and modellings carried out by Davies (1976), and by Saito, Doms, Schaettler and Steppler (1998), shows that relaxation zone with a depth of 5 – 6 grid-points is already sufficient for the **NH** buoyancy wave absorption.

IV. CONCLUSIONS

A preliminary version of the nonhydrostatic HIRLAM is close to be completed. For completion the lateral BC should be modified. The model employs Eulerian integration scheme and enables computation of additional tendencies due to nonhydrostatic accelerations, which are caused by the orography and by inertial forcing. The model is quasi-planar : Earths sphericity is treated as a small perturbation to the plain geometry, which restricts the domain of integration to be less than 1500 – 2000 km.

Actual problems (as seen in October 1998):

– Modification of lateral boundary conditions and implementation of the Davies relaxation scheme.

– Implementation of the spherical geometry. Requires a new solution scheme for the elliptic equation, which, similarly to the existing Helmholtz-scheme of the **HS** HIRLAM, employs eigenvector technique in the vertical dimension.

– Resolution enlargement ($\Delta X = 11$ km $\rightarrow \Delta X = 5.5, 2.25$ km).

– Tests on larger grids.

– Development of the semi-implicit **NH** scheme.

– Implementation of **NH** algorithm with the Lagrangian time-integration scheme.

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